Guide to Estimating Prestress Loss
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Guide to Estimating Prestress Loss

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This guide is intended for estimation of prestress losses in concrete structures. Methods presented include lump sum, simplified approaches addressing individual source of loss, and additional estimation methods. They address losses in pretensioned and post-tensioned members, including bonded, unbonded, and external tendons. Note that these estimation methods have not been evaluated for relative merits. A discussion of the variability of prestress losses caused by the variability in concrete properties is also presented. Several example problems are included.

Keywords: creep; friction; post-tensioning; prestress loss; prestressed concrete; relaxation; shrinkage.
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CHAPTER 1—INTRODUCTION

1.1—Introduction
   Estimating prestress loss at any given time during the life of a prestressed concrete member is a complex issue. In pretensioned and post-tensioned members, applying prestressing force causes shortening of the concrete member that, in turn, causes a loss of tendon stress. Over time, concrete creep, concrete shrinkage, and steel relaxation result in additional reductions of tendon stress. In post-tensioned members, losses occur during the stressing operation due to friction between the tendon and sheathing or duct, which is caused by the intended and unintended tendon curvature. There are also losses due to seating of the wedges or nuts as the jacking force is transferred into the anchorage device. These and other sources of prestress loss are examined by the licensed design professional to get an estimate of the total prestress loss and resulting effective prestressing force.

   Losses have inherent variability due to variations of material properties and environmental and curing conditions. Some losses may affect others. Time-dependent concrete properties are particularly difficult to estimate accurately, so losses due to creep and shrinkage are expected to be variable. Friction between the tendon and sheathing or duct, movement of wedges within the anchorage device, and modulus of elasticity of concrete are also variables. The variability within each component and the interdependence among the components make it understandable that studies comparing measured prestress losses to predictions have shown that accurate and consistent calculation of prestress loss is difficult to achieve.

   The best effort to calculate prestress loss is only an estimate and, therefore, the licensed design professional should consider the consequences of actual losses being higher or lower than the estimated value. Estimation of prestress loss is an important factor for evaluating the serviceability of all types of prestressed members and the calculation of flexural strength of members with unbonded tendons. The estimation of prestress loss, however, is not a significant factor in determination of flexural strength of bonded prestressed members. When computing the shear strength of prestressed members with little or no transverse reinforcement, a conservative estimate of the effective prestressing force is warranted.

1.2—Scope
   ACI 318-11 requires that the design of prestressed concrete members allow for prestress loss; however, the required level of detail for calculating losses is unspecified. The friction loss provisions for post-tensioned construction that first appeared in ACI 318-63 were removed from ACI 318-11. Although ACI 318-11 Commentary indicates that the lump sum method is obsolete, the licensed design professional’s requirement to choose a method to compute losses remains. This guide is intended to aid the designer in this choice by providing an overview of the various methods available.

   Many participants in the design and construction process need information on prestress losses. The licensed design professional, precasters, and post-tensioners all need an understanding of, and method to estimate, aspects of losses. To which entity is responsible for calculation of each type of loss has to be clearly defined in the contract documents.

   Total losses, Δf_{fr}, are losses due to friction and seating Δf_{fr,SE}, elastic shortening Δf_{ES}, creep of concrete Δf_{CRE}, shrinkage of concrete Δf_{SH}, and relaxation of tendons Δf_{REL}. This can be expressed as Eq. (1.2)
This guide presents background information and methods to calculate each type of loss. Following the introduction and a list of notation and definitions, Chapter 3 includes a historical account of the lump sum method, currently recommended values for preliminary design, and a summary of losses that have been measured in field and laboratory studies.

Chapter 4 discusses the different types of initial losses and addresses the differences between pretensioned and post-tensioned members.

Chapter 5 presents a simplified approach to estimate long-term losses due to creep, shrinkage, and relaxation for pretensioned and post-tensioned concrete members.

Detailed approaches to estimate long-term losses are presented in Chapter 6, which also addresses changes in prestressing force caused by differential shrinkage and hydration of the concrete deck in composite members. The approaches can be used for pretensioned or post-tensioned members.

Chapter 7 discusses the variability of prestress loss calculations caused by concrete material properties, including compressive strength at transfer, modulus of elasticity, and creep and shrinkage.

Chapter 8 presents example problems and compares solutions from different methods.

13—Historical development

The concept of prestressing concrete dates back to the late 1800s (Naaman 2012). The performance of early prestressed concrete structures was adversely affected by time-dependent strains in the concrete—for example, creep and shrinkage, which were nearly as large as the initial steel strain due to prestressing. Before 1940, the initial steel strain induced by prestressing was limited by the low yield strength of steel. French engineer Eugene Freyssinet recognized the significance of prestress losses and the need for steels with high yield strength for prestressed applications. By 1945, higher strength steel became available, making it possible to produce the initial prestressing strain large enough so that the time-dependent strains developed in the concrete would not overcome the initial prestressing strain. As a result, the remaining prestressing force in the steel would be sufficiently large to be effective.

Prestress losses were first addressed by ACI 318 in 1963. Although the provisions catalogued the different causes of prestress loss, they only provided specific instruction on determining friction losses. These code provisions were based on an earlier committee publication that provided similar, slightly more detailed guidance on prestress loss (ACI-ASCE Committee 323 1958).

In the 1970s, the Precast/Prestressed Concrete Institute (PCI Committee on Prestress Losses 1975) and Zia et al. (1979) provided more detailed methods to estimate prestress losses. Since the 1970s, others have developed methods to estimate prestress losses (Tadros et al. 2003; Seguirant and Anderson 1985; Youakim et al. 2007; Garber et al. 2013).

Gilbert and Ranzi (2011) and Branson (1977) provide general approaches to the calculation of a variety of time-dependent effects in concrete structures, including prestress losses. Computer programs have been developed to perform the tedious calculations required for stepwise analyses of prestress loss. However, due to the inherent uncertainties associated with material properties, construction practices, and in-service conditions, even the most refined calculations result in prestress loss predictions that differ from measured values.

1.3.1 Currently available guidance on estimating prestress losses—For pretensioned building products, the PCI Design Handbook (PCI 2010) presents a method to estimate prestress losses based on the method developed by Zia et al. (1979). This method is widely used for building structures and is referenced in the R18.6.1 commentary of ACI 318-11, and presented in this guide in Chapter 5. For bridge beams, the “AASHTO LRFD Bridge Design Specification” (AASHTO 2012) presents two methods. One is an approximate method and the other a refined method based on several parameters to estimate prestress losses. The refined method could be applied to building products as well. These methods are presented in Chapters 5 and 6.

1.4—Guide organization and use

This guide presents a variety of approaches for estimating prestress losses in pretensioned and post-tensioned members. This section identifies relevant sections of interest in the guide, depending on member type (pretensioned or post-tensioned) and level of effort (lump sum, simplified, or detailed). The lump sum method is only recommended for preliminary designs. The simplified method is appropriate for most typical designs. Detailed methods are most often used for more complex structures, which may have staged construction and prestressing operations.

1.4.1 Pretensioned members—Losses for pretensioned members are classified as initial or long-term. One group of initial loss occurs during stressing and before transfer of prestress due to friction, seating losses, and temperature effects. It is the precaster’s responsibility to understand the magnitude of these losses and account for them to provide the specified strand stress before transfer. Information on these types of losses is found in:

(a) Anchorage seating—4.2.1
(b) Form and abutment deformations—4.2.2
(c) Thermal effects—4.2.4
(d) Steel relaxation—4.2.5

Another initial loss is elastic shortening of the member that occurs at the time of transfer. As the prestress force is transferred to concrete, the member shortens. The steel and concrete are fully bonded, so the steel shortens with the concrete. This shortening causes a loss in stress in the prestressing steel, known as the elastic shortening loss, which should be accounted for by the designer. Long-term losses occur due to concrete creep and shrinkage and prestressing steel relaxation. Other changes of tendon force can occur due to temperature effects and external loads placed on the member at the time of casting or in service.
1.4.1.1 Pretensioned members/lump sum method—The lump sum method presented in Chapter 3 is often used for preliminary design. The values presented in sources referenced in Chapter 3 typically include all losses, both initial and long-term.

1.4.1.2 Pretensioned members/simplified method—The simplified method is a commonly employed approach to estimate prestress losses in typical pretensioned members. The designer needs to calculate four components of loss and add them together for the total prestress loss. Components and applicable sections are:

(a) Elastic shortening—4.3.2
(b) Creep—5.2
(c) Shrinkage—5.3
(d) Relaxation—5.4

1.4.1.3 Pretensioned members/detailed method—This guide provides information on more detailed methods of prestress loss estimation. Two alternate methods for a more detailed calculation of elastic shortening losses in pretensioned members are:

(a) Transformed section method—4.3.1
(b) Iterative gross section method with iteration—4.3.3

More detailed approaches to calculate long-term losses are presented in Chapter 6. These methods are used with a variety of creep and shrinkage models, as opposed to the simplified method, which uses a single model. Detailed methods also allow the designer to consider the influence of a cast-in-place composite deck if needed, whereas the simplified method only accounts for the weight of the deck, but not other factors such as differential shrinkage and internal stress redistributions between the beam and the deck, if acting compositely. The detailed methods are:

(a) AASHTO LRFD refined method (AASHTO 2012)—6.3.2
(b) General age-adjusted effective modulus method (Menn 1990)—6.3.3
(c) Incremental time-step method (Nilson 1987)—6.4

Chapter 6 (6.6) also provides information on the approximation of changes due to thermal effects of deck casting.

1.4.2 Post-tensioned members—Several approaches can be used to approximate prestress losses in post-tensioned members. Initial losses encompass all prestress loss during the stressing operation, including friction due to wobble and curvature, seating losses, and elastic shortening losses. The estimation of long-term losses for bonded post-tensioned members is essentially the same as for pretensioned members. Calculation of long-term losses in unbonded post-tensioned members is different, because losses are related to the overall change in tendon length, rather than the change in strain at a specific section.

1.4.2.1 Post-tensioned members/lump sum method—The lump sum method, presented in Chapter 3, is typically used only for preliminary designs. Before adopting a value for use in preliminary design, the licensed design professional should determine if the presented value includes friction and seating losses.

1.4.2.2 Post-tensioned members/simplified method—The simplified method can be used to estimate prestress losses in typical post-tensioned members. The designer needs to calculate five components of loss and add them together for the total prestress loss. Components and applicable sections are:

(a) Friction and seating loss—4.4
(b) Elastic shortening loss—4.5
(c) Creep loss—5.2.1 (bonded)
(d) Creep loss—5.2.2 (unbonded)
(e) Shrinkage loss—5.3
(f) Relaxation loss—5.4

Note that elastic shortening losses only occur in post-tensioned members with multiple tendons when the tendons are stressed sequentially. Tendons stressed first will incur losses as the concrete shortens due to the stressing of subsequent tendons.

1.4.2.3 Post-tensioned members/detailed methods—As with pretensioned members, long-term prestress loss in post-tensioned members is estimated using more detailed methods presented in Chapter 6, with a detailed description in 1.4.1.3. Initial losses are calculated per 4.3.3 and 4.4.
ɛ_e = eccentricity of centroid of tendons with respect to the centroid of the f_or transformed concrete at the cross section considered, in. (mm)

E(t) = modulus of elasticity at any time t, psi (MPa)

E_r = modulus of elasticity of concrete, psi (MPa)

E_r’ = effective modulus of elasticity of concrete, psi (MPa)

E_r” = age-adjusted effective modulus of elasticity of concrete, psi (MPa)

E_cd = modulus of elasticity of the composite deck, psi (MPa)

E_ci = modulus of elasticity of concrete at time of application of prestress, psi (MPa)

E_{ci}(t) = modulus of elasticity of concrete at time t, psi (MPa)

E_p = modulus of elasticity of the prestressing steel, psi (MPa)

f_{anchor} = strand stress at the anchorage device after seating, psi (MPa)

f_c = concrete compressive stress, psi (MPa)

f_c’ = specified compressive strength of concrete, psi (MPa)

f_cd = concrete stress at center of gravity of prestressing force due to all superimposed permanent loads that are applied to the member after it has been prestressed, psi (MPa)

f_c’ = specified compressive strength of concrete at transfer of prestress, psi (MPa)

f_c = concrete compressive stress immediately after transfer at fiber under investigation, psi (MPa)

f_{c,cr} = net compressive concrete stress at center of gravity of prestressing force immediately after the prestress has been applied to the concrete, psi (MPa)

f_{crs} = average compressive concrete stress at the center of gravity of the tendons immediately after the prestress has been applied to the concrete, psi (MPa)

f_{p,cr} = concrete stress at center of gravity of prestressing force due to all prestress and applied loads, psi (MPa)

f_{p,cr} = tendon stress at nonstressing end, psi (MPa)

f_{p,cr} = jacking stress, psi (MPa)

f_c = stress in prestressing steel at a distance L from jacking end, psi (MPa)

f_c/2 = stress in prestressing steel at a distance L/2 from the jacking end, psi (MPa)

f_{max} = maximum stress in the prestressing steel along the tendon length, psi (MPa)

f_{pre} = stress in prestressing steel immediately before transfer, psi (MPa)

f_p = prestressing steel stress immediately following transfer, psi (MPa)

f_{ps} = prestressing steel stress after jacking and seating, psi (MPa)

f_{ps}(t) = stress in prestressing steel at time t, psi (MPa)

f_r = stress in prestressing steel immediately after transfer, psi (MPa)

f_m = specified tensile strength of prestressing steel, psi (MPa)

f_y = specified yield strength of prestressing steel, psi (MPa)

f_a = stress in prestressing steel at a distance x from the jacking end, psi (MPa)

I_c = moment of inertia of the composite cross section, in.4 (mm4)

I_d = moment of inertia of the deck, in.4 (mm4)

I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, in.4 (mm4)

I_{cr} = moment of inertia of transformed concrete section about centroidal axis, including reinforcement, in.4 (mm4)

J = factor in calculation of prestress loss due to relaxation according to the PCI Design Handbook (PCI 2010) method

k = wobble friction coefficient per unit length of tendon, per ft (per m)

k_f = factor for the effect of concrete strength

k_d = humidity factor for creep

k_s = factor for the effect of volume-to-surface ratio

k_t = time development factor

K_{cr} = modification factor in PCI Design Handbook (PCI 2010) method in calculation of concrete stress due to prestressing force immediately after the prestress has been applied to the concrete

K_{cr} = coefficient in the PCI Design Handbook (PCI 2010) method to account for loss due to creep

K_{sg} = transformed section coefficient

K_{sp} = factor in calculation of elastic shortening losses in Zia et al. (1979) method

K_{sd} = section modification factor from AASHTO (2012) prestress loss method

K_{se} = factor in calculation of prestress loss due to relaxation in PCI Design Handbook (PCI 2010) method

K_{sb} = factor in calculation of prestress losses due to shrinkage in PCI Design Handbook (PCI 2010) method

L = length from anchorage to anchorage, ft or in. (m or mm)

L_{beam} = length of beam, ft or in. (m or mm)

L_{free} = length of strand outside of beam, ft or in. (m or mm)

M = bending moment experienced by cross section immediately after transfer (usually due to self-weight), in.-lb (N-mm)

M_{b,cr} = initial creep-producing moment in the girder, in.-lb (N-mm)

M_{deck} = moment in beam due to the weight of the deck, in.-lb (N-mm)

M_{b,cr} = bending moment due to dead weight of prestressed member and any other permanent loads in place at the time of prestressing, in.-lb (N-mm)

M_{cr} = moment due to all superimposed permanent loads applied after prestressing, in.-lb (N-mm)

n_f = modular ratio; modulus of prestressing steel divided by modulus of concrete

N = number of sequentially stressed tendons

N_{b,cr} = initial creep-producing force in the girder, lb (N)

P = applied tension force, lb (N)

P_{avg} = average force in the tendon, lb (N)

P_i = initial prestress force after anchorage seating loss, lb (N)
\[ P_t = \text{maximum prestress force during jacking operation, lb (N)} \]
\[ P_p = \text{prestress force before release, lb (N)} \]
\[ P_{shd} = \text{force to fully restrain the shrinkage of the composite deck, lb (N)} \]
\[ RH = \text{average ambient relative humidity in percent} \]
\[ s = \text{slope of stress in prestress versus distance line, psi/ft (MPa/m)} \]
\[ S_b = \text{section modulus with respect to the bottom fiber of the beam, in.}^3 (\text{mm}^3) \]
\[ t = \text{time under consideration from time of release, days} \]
\[ t_c = \text{age of concrete, days} \]
\[ t_s = \text{time since end of cure to time of deck placement, days} \]
\[ t_f = \text{final time under consideration from time of release, days} \]
\[ t_1 = \text{1-day steam cured} \]
\[ t_2 = \text{time of initial loading from time of release, days} \]
\[ T_s = \text{prestressing force at stressing end, lb (N)} \]
\[ T_c = \text{prestressing force at point } x, \text{ lb (N)} \]
\[ T(y) = \text{temperature of cross section at distance } y \text{ from centroid, } ^\circ \text{F (°C)} \]
\[ V/S = \text{ratio of volume to surface area of concrete element, in.}^3/\text{in.}^2 (\text{mm}^3/\text{mm}^2) \]
\[ w_c = \text{unit weight of normalweight concrete or equilibrium density of lightweight concrete, lb/ft}^3 (\text{kg/m}^3) \]
\[ w_t = \text{weight of topping, lb/ft}^2 (\text{N/m}^2) \]
\[ x = \text{length of tendon from stressing end to point } x, \text{ ft or in. (m or mm)} \]
\[ x_s = \text{length influenced by anchor set, ft or in. (m or mm)} \]
\[ y = \text{distance from centroid of cross section to location under consideration, in. (mm)} \]
\[ y_{bot} = \text{the distance from the centroid of the composite section to the bottom of the section, in. (mm)} \]
\[ y_{top} = \text{distance from centroid of transformed section to concrete fiber under investigation, in. (mm)} \]
\[ \alpha = \text{total angular change from jacking end to point } x, \text{ radians} \]
\[ a_c = \text{coefficient of thermal expansion of concrete, } ^\circ \text{F (°C)} \]
\[ a_{ps} = \text{coefficient of thermal expansion of prestress, } ^\circ \text{F (°C)} \]
\[ \beta = \text{constant such that } \beta = (28 - a)/28 \]
\[ \gamma_h = \text{correction factor for ambient relative humidity} \]
\[ \gamma_m = \text{correction factor for specified concrete compressive strength at transfer} \]
\[ \Delta = \text{tendon elongation, in. (mm)} \]
\[ \Delta f_c = \text{change in concrete stress at the center of gravity of the prestressing force due to the differential shrinkage force, psi (MPa)} \]
\[ \Delta f_{cp} = \text{change in concrete stress at the center of gravity of the prestressing force due to application of superimposed load, psi (MPa)} \]
\[ \Delta f_{shd} = \text{change in stress due to anchor set, psi (MPa)} \]
\[ \Delta f_{CD} = \text{prestress loss due to creep of girder concrete between time of deck placement and final time, psi (MPa)} \]
\[ \Delta f_{CR} = \text{prestress loss due to creep of girder concrete during transfer and deck placement, psi (MPa); (AASHTO 2012)} \]
\[ \Delta f_{ELG} = \text{increase in prestress (elastic gain) due to addition of superimposed permanent loads, psi (MPa)} \]
\[ \Delta f_{ES} = \text{prestress loss due to elastic shortening, psi (MPa)} \]
\[ \Delta f_{FS} = \text{prestress loss due to friction and seating, psi (MPa)} \]
\[ \Delta f_{LT} = \text{long-term prestress loss, psi (MPa)} \]
\[ \Delta f_{R1} = \text{prestress loss due to relaxation of prestressing strands between time of transfer and deck placement, psi (MPa)} \]
\[ \Delta f_{R2} = \text{prestress loss due to relaxation of prestressing strands in composite section between time of deck placement and final time, psi (MPa)} \]
\[ \Delta f_{RSE} = \text{prestress loss due to relaxation, psi (MPa)} \]
\[ \Delta f_{RSD} = \text{prestress loss due to shrinkage of girder concrete between time of deck placement and final time, psi (MPa)} \]
\[ \Delta f_{RSH} = \text{prestress loss due to shrinkage, psi (MPa)} \]
\[ \Delta f_{RGR} = \text{prestress loss due to shrinkage of girder concrete between transfer and deck placement, psi (MPa)} \]
\[ \Delta f_{RSS} = \text{prestress gain due to shrinkage of deck in composite section, psi (MPa)} \]
\[ \Delta f_{LT} = \text{total prestress loss, psi (MPa)} \]
\[ \Delta e_{pk} = \text{elongation of prestressing force due to the differential anchor set} \]
\[ \Delta e_{pR} = \text{change in moment in the beam, in.-lb (N-mm)} \]
\[ \Delta M_d = \text{change in moment in the deck, in.-lb (N-mm)} \]
\[ \Delta N_b = \text{change in force in the beam, lb (N)} \]
\[ \Delta N_d = \text{change in force in the deck, lb (N)} \]
\[ \Delta N_{ps} = \text{change in prestress force, lb (N)} \]
\[ \Delta N_{rel} = \text{change in prestress force due to relaxation (no associated strain), lb (N)} \]
\[ \Delta N_{redo} = \text{change in strain in the beam at the gross section centroid} \]
\[ \Delta e_c = \text{change in strain in the beam at the center of gravity of the prestressing force} \]
\[ \Delta e_{cr} = \text{change in strain in concrete due to creep} \]
\[ \Delta e_d = \text{change in strain at the centroid of the deck} \]
\[ \Delta e_{ps} = \text{change in strain in prestressing steel} \]
\[ \Delta e_{redo} = \text{change in strain in prestressed reinforcement due to anchor set} \]
\[ \Delta e_{free} = \text{change in strain in free length of prestressing steel} \]
\[ \Delta e_{d0} = \text{change in strain in the beam due to shrinkage} \]
\[ \Delta e_{d0}(t_s) = \text{shrinkage strain in the beam concrete at the time the deck is placed} \]
\[ \Delta e_{d0}(t_f) = \text{shrinkage strain in the beam concrete at the final time considered} \]
\[ \Delta e_d = \text{change in strain in the deck steel} \]
\[ \Delta e_{cr} = \text{change in curvature, in./in. (mm/mm)} \]
\[ \Delta e_{ps} = \text{change in stress at time } t_s, \text{ psi (MPa)} \]
The chapter summarizes lump-sum values from past codes and proportional limit of the material. In addition, data from the Bureau of Public Roads (1954), the Precast/Prestressed Concrete Institute (PCI) published building code requirements for prestressed concrete (Structural Engineers Association of Northern California 1959) specifying the following lump-sum values for estimating prestress losses, excluding friction prestressing steel loss: pretensioning = 25,000 to 35,000 psi (172 to 241 MPa); and post-tensioning = 15,000 to 25,000 psi (103 to 172 MPa).

In published discussions, Abeles (1958) argued that these recommended lump-sum values should be eliminated or increased to allow designers to assess the losses for specific conditions. If the lump-sum values were to remain, he suggested the following ranges of values: pretensioning = 30,000 to 40,000 psi (207 to 276 MPa); and post-tensioning = 20,000 to 30,000 psi (138 to 207 MPa). Similarly, the Precast/Prestressed Concrete Institute (PCI) Committee on Prestress Losses (1975) and PCI Committee on Prestress Losses (1979). Although ACI 318-11 does not explicitly prohibit use of lump-sum losses, the commentary is clear in warning that previously cited lump-sum losses are considered obsolete.
3.3—Industry practice

The Post-Tensioning Institute’s manual (PTI 2006) provides a discussion of long-term losses and a detailed discussion of friction losses, which are addressed elsewhere in this guide, including 4.4.2. While PTI refers to Zia et al. (1979), it is noted that precise determination of losses in post-tensioned members is not critical. The discussion is closed with the statement, “A 100% variance in the estimate for total long-term losses generally results in less than a 10% difference in stress in the prestressing steel at nominal strength.” Kelley (2000) indicates that for approximating prestress losses, tendons are typically assumed to be stressed to 0.8$f_{pm}$ for ASTM A416/A416M Grade 270 (1860 MPa), this equates to a tendon stress of 216,000 psi (1490 MPa). In addition, it is typically assumed that the average tendon stress along a tendon after seating is 0.7$f_{pu}$. Based on an assumed final effective prestress of 175,000 psi (1210 MPa), the total prestress loss is 41,000 psi (282 MPa), or 19 percent of the initial jacking stress. This results in a force of 26,800 lb (119,000 N), often rounded to 27,000 lb (120,100 N), for a single 0.5 in. (12.7 mm) diameter strand.

Before low-relaxation strand gained popularity in the 1970s, use of stress-relieved strand resulted in higher losses due to relaxation of the prestressing strand. At that time, a total loss of $52,000$ to $57,000$ psi (359 to 393 MPa) was typically assumed, resulting in a final effective prestress force of approximately 24,000 to 25,000 lb (106,800 to 112,200 N) per strand.

The PCI Design Handbook (PCI 2010) indicates that losses have no effect on the ultimate strength of a flexural component unless the tendon is unbonded or has an effective prestress of less than 0.5$f_{pu}$. In addition, the handbook indicates that overestimated or underestimated losses may impact service limit states such as camber, deflection, or cracking. PCI (2010) also provides an estimate of total losses for typical concrete components:

(a) Normalweight concrete: 25,000 to 50,000 psi (172 to 345 MPa) (12 percent to 25 percent)

(b) Sand lightweight concrete: 30,000 to 55,000 psi (207 to 379 MPa) (15 percent to 27 percent)

These values are presented as total losses for typical components. Losses in terms of the percentage of an assumed initial jacking force of 0.75$f_{pu}$, are also given. For refined estimation of losses, PCI (2010) refers to Zia et al. (1979).

3.4—Measured losses

Measurement of prestressing losses is challenging because it requires that concrete strain be determined from the time when the prestress is transferred until the end of the measurement period. This typically excludes use of bonded electrical resistance foil gauges because of drift due to temperature change and lack of durability when exposed to concrete and the surrounding environment. Consequently, losses are typically measured using vibrating wire strain gauges embedded in concrete or mechanical gauges, which measure relative movement of gauge points adhered to the concrete surface. These movements are used to calculate average concrete strain between the points and to track change in concrete strain over time. If perfect bond is assumed between prestressing steel and concrete, then change in concrete strain is equal to change in steel strain.

If steel strain has been tracked from the initial value, which is typically the initial stressing, then the measured strain is used to compute the change in steel stress due to change in steel strain. The conversion from strain to stress requires that the elastic modulus of the prestressing steel ($E_p$) be known. While determining $E_p$ for single wires and bars is relatively straightforward, the nature of the seven-wire strand makes accurate and precise measurement of $E_p$ more difficult. Additionally, strain measurement methods cannot account for time-dependent losses due to relaxation. For low-relaxation prestressing strand, losses due to relaxation are usually minor relative to other time-dependent losses. For other materials more susceptible to relaxation, a force-measuring device is required, which is not typically a practical alternative. The measured change in steel stress is equal to the prestress loss from the time when the initial measurement was taken to when the measurements were terminated. The ultimate value of long-term losses is practically unattainable due to the time over which measurements are taken. Fortunately, most of the long-term losses occur in the first 6 months after prestress transfer. Consequently, relatively short-term measurements of losses, which range from 6 months to 2 years, can provide useful information regarding ultimate long-term losses.

Measurement techniques described previously require that instrumentation be installed during fabrication; readings are then taken before prestress transfer and regularly during the monitoring period. For in-service girders not previously instrumented, measurement of crack opening during loading may be used to estimate the effective prestress (Pessiki et al. 1996). While not as reliable or accurate as vibrating wire or mechanical gauges, it does provide a method for estimating the effective prestress in noninstrumented, in-service girders.

Table 3.4 presents measured losses reported in the literature. Nearly all data are from measurements on precast, pretensioned, I-shaped bridge girders. Spans range from less than 10 ft (3 m) to more than 270 ft (82.3 m). In addition, concrete compressive strengths vary from 4000 psi (27.6 MPa) to more than 15,000 psi (103 MPa) with an average of approximately 9000 psi (62.0 MPa). While little data are available on prestressed concrete elements used specifically in building construction, all or part of the bridge data presented herein is useful as a benchmark against which, for future evaluation, either bridge or building components can be compared.

Measured total losses were gathered from each reference and include instantaneous and long-term losses for the associated measurement period indicated in the table. No adjustments for final total losses have been made. Losses are reported in terms of stress in the prestressing steel and as a percentage of an assumed initial prestress force equal to 0.75$f_{pu}$. Overall average prestress losses were approximately 40,700 psi (281 MPa), or 21 percent, with a coefficient of variation (COV) of 40 percent that indicates a wide data scatter. While the data are quite scattered, average total loss...
Table 3.4—Measured losses from literature

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<th>Span, ft</th>
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<th>Instrumentation</th>
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is within the range of estimated lump-sum losses of 25,000 to 50,000 psi (172 to 345 MPa) given in PCI (2010).

Wide scatter of these measurements is partially due to variation in measurement techniques and the length of time over which measurements were taken. It is also an indication that variations in material properties, design characteristics, and exposure conditions ultimately lead to prestress losses that are naturally variable and almost impossible to predict with either great precision or accuracy.

**CHAPTER 4—INITIAL LOSSES**

4.1—**Scope**

Chapter 4 addresses initial losses, sometimes called short-term, instantaneous, or immediate losses, for pretensioned and post-tensioned members. These losses include friction and seating losses during tensioning operations, and elastic shortening losses that occur when the prestressing force is transferred to concrete. Instantaneous changes in prestress force caused by applied loads are discussed later in this chapter.

Friction and seating losses are discussed for pretensioned and post-tensioned members. Basic friction concepts due to curvature and wobble are covered. The friction and seating losses that occur during the stressing operations in a precast plant are the responsibility of the prestressor. Precasters understand the sources and magnitudes of the losses and compensate for them during initial stressing, as described in 4.2. The designer does not typically need to consider these losses for prestressed members. For post-tensioned members, friction and seating loss calculations are required. The differences between unbonded, bonded, single-strand, and multi-strand tendons are shown in 4.4, along with recommended friction and wobble coefficients specific to the type of duct used. Seating losses for wedge anchorages and bar nuts are also discussed in 4.4.
Also addressed are changes in prestress force that result from elastic deformations of a prestressed concrete member. The discussion is limited to ranges of member behavior in which the concrete and prestressed and nonprestressed reinforcement exhibit linearly elastic material behavior. For the purposes of estimating initial prestress losses, this is assumed when the member is designed to satisfy the ACI 318 prestressing steel stress limits and concrete stress limits at prestress transfer.

Sections 4.2 and 4.3 discuss losses before and at transfer in pretensioned members, while 4.4 and 4.5 present these losses for post-tensioned members. Section 4.6 discusses subsequent changes to the effective prestress due to externally applied loads.

4.2—Pretensioning losses before transfer

In pretensioned construction, the prestressing takes place in a precasting facility and the producer is usually responsible for providing the correct prestress force immediately before transfer. The licensed design professional has only limited information regarding fabrication procedures.

During stressing operations and concrete curing, strand stress varies. Sources of stress variation are discussed herein, along with strand elongation, which is typically measured to ensure that the strands have the expected level of stress before prestress transfer.

4.2.1 Anchor set—Anchor set is the movement of strands within the anchors as the strands are stressed and anchored. Wedges in the anchorage typically grip the strand and move forward slightly as the jaws seat in the tapered wedge cavity.

Stressing-end anchor set occurs during single-strand tensioning as the final jacking force is transferred from the stressing device to the anchorage. A measurable decrease in strand elongation occurs as the wedges move into the tapered wedge cavity, and the strand stress decreases accordingly. The change in prestress due to anchor set \( \Delta P_{\text{psd}} \) is computed

\[
\Delta P_{\text{psd}} = E_p \Delta \varepsilon_{\text{psd}} = E_p \frac{\Delta s}{L}
\]  

(4.2.1)

where \( E_p \) is modulus of elasticity of prestressed steel, psi (MPa); \( \Delta \varepsilon_{\text{psd}} \) is change in strain in prestressed reinforcement due to anchor set, in./in. (mm/mm); \( \Delta s \) is anchor set, in. (mm); and \( L \) is strand length anchorage to anchorage, ft or in. (m or mm).

The decrease in elongation associated with this anchor set typically ranges from 1/8 to 3/8 in. (3 to 10 mm). For a specific anchor set value, the magnitude of the prestress loss due to anchor set decreases with increasing length between anchorages. In some pretensioning operations, the prestressing beds are long enough, which is commonly 300 to 500 ft (90 to 150 m), that the anchorage seating losses are relatively small. Seating losses, however, will be larger in shorter prestressing beds. Additional forces are applied during stressing to compensate for the anticipated seating loss effect.

In the case of multiple-strand tensioning operations, the stressing-end wedges are seated in the anchorage as the force is applied. Therefore, no loss of prestress force occurs relative to the measured value. Likewise, seating loss for fixed-end anchorages and splicing anchorages also occurs during tensioning, and there is no loss in prestress force relative to the measured value.

4.2.2 Form or abutment deformations—Two strand-restraint systems—self-stressing forms and fixed-abutment beds—are considered separately in this section.

4.2.2.1 Self-stressing forms—Self-stressing steel forms have strand anchorages bearing against plates that transfer the jacking force directly to the forms. Therefore, form shortening occurs during the tensioning operation. Figure 4.2.2.1a shows a self-stressing form.

The form shortens incrementally as each strand is stressed. This shortening, which is illustrated in Fig. 4.2.2.1b, might not be uniform for each strand due to flexure of the form, stressing sequence, and friction between the form and its supports.

If strands are stressed sequentially, the stress in each strand will decrease when subsequent strands are stressed. The amount of form shortening that takes place should be measured periodically for different numbers of strands in common configurations. The initial tensioning force may be increased to compensate for the anticipated form-shortening effect on the average strand.
Fig. 4.2.2.2a—Fixed abutments.

Abutment deflection at center of gravity of strand

Center of gravity of strand

Measured elongation = strand elongation and abutment deflection

Fig. 4.2.2.2b—Fixed-abutment deformations.

4.2.2.2 Fixed-abutment beds—Abutment movement or movement of anchorages occurs during tensioning of fixed-abutment beds. Figure 4.2.2.2a shows two fixed abutments. Even large, well-designed abutments will rotate and deflect slightly when loaded, as illustrated in Fig. 4.2.2.2b. Although the movement is small, it should be monitored and evaluated for strand patterns commonly used. The influence of this movement decreases with increasing bed length. As with self-stressing forms, if strands are tensioned sequentially, the tensioning force may need to be increased to compensate for the effects of abutment movement on previously stressed strands.

4.2.3 Elongation calculation and correction—A prestressing strand’s length increases in proportion to the applied tension. This increase in length is called elongation. Measured strand elongations are typically compared to expected values to verify the prestress force before transfer. Elongation is commonly discussed using three terms: basic, gross, and net.

The basic elongation $e_b$ is the length the strand is expected to elongate based on the following factors only:

- Applied tension force $P$, lb (N)
- Strand length from anchorage to anchorage, $L$, in. (mm)
- Strand cross-sectional area $A_p$, in.$^2$ (mm$^2$)
- Strand modulus of elasticity, $E_p$, psi (MPa)

ACI 318-11 and AASHTO (2012) design specification stress limits ensure that the prestressing reinforcement remains in the elastic range during tensioning. Therefore, Eq. (4.2.3) for calculating the basic elongation of straight strands is

$$e_s = \frac{PL}{A_p E_p}$$  \hspace{1cm} (4.2.3)

A more complex formulation may be warranted for harped (draped) strands.

In a pretensioning operation, an initial force removes slack from the strands during installation, and the elongation measurements follow after the initial force is applied. The applied tension force for elongation computation, therefore, is determined by subtracting the initial force from the final force measured during tensioning. Strand length is the length between the stressing plates on which the strand chucks bear at each end of the casting bed. Strand cross-sectional area and modulus of elasticity are taken from the strand manufacturer’s mill report for each pack of strand.

Strand area and modulus of elasticity may vary for different heat numbers, which identify specific production runs as well as manufacturers. Average values of area and modulus of elasticity for different packs of strand may be used for elongation calculations, provided they vary less than 2.5 percent for all strands in use. Otherwise, the elongation is recomputed using the actual strand properties.

Elongation is typically measured as the displacement of a mark on the strand relative to a reference point on the stressing-end prestressing chuck or anchor. The elongation measured before stressing-end anchorage seating is known as gross elongation. Net elongation is the elongation measured after anchorage seating. Gross and net elongation are used to check strand tension before and after seating loss, respectively.

Because elongation is measured at the stressing-end anchorage, measured elongations may include operational components that do not reflect actual strand elongation, such as strand seating displacements at fixed-end or splicing anchorages, or deformation of the abutments or self-stressing forms during stressing. The characteristics of each pretensioning setup are determined and appropriate compensations made when calculating the expected elongation values.

4.2.4 Thermal effects

4.2.4.1 Stress changes from thermal effects—When strands are anchored between fixed abutments, changes in strand temperature after tensioning change the strand force. Strands tensioned on a cold morning will lose force when warmer concrete raises the strand temperature because the length between anchorages is not affected by the concrete temperature. The opposite effect could occur for strands tensioned on a hot afternoon. The strand stress can decrease approximately 1000 psi (6.89 MPa) for a differential temperature increase of 5°F (2.8°C). Thermal stress changes will not occur with self-stressing forms because the form will expand and contract with temperature changes similar to the strand.

Several researchers have noted that the heat of hydration during curing of a pretensioned beam may affect the force in the prestressing steel (Gross 1998; Barr et al. 2000).
Because the strands are anchored in place at the abutments, they have a constant length from the time they are anchored until they are released. If the strands are under a constant strain, are heated by the hydrating concrete, they will experience a loss in stress. The loss is difficult to quantify because it depends on when the concrete and steel begin to bond. After the concrete has reached its peak temperature, the concrete and steel begin to cool. However, because the strand contraction is restrained by the abutments, increased tensile stresses will develop in the strand, and tension may develop in the concrete beam. The gain in tension in the strand outside the beam upon cooling is not the same as the loss of tension will develop in the strand, and tension may develop in the concrete beam. The gain in tension in the strand outside the beam upon cooling is not the same as the loss of tension during heating, because the steel is bonded to the concrete. However, all changes in stresses that occur due to temperature changes after the concrete and steel are fully bonded are recovered after the strands are cut. After transfer of the prestress, temperature changes will have a small influence on the prestress force because the concrete and steel will expand and contract as a unit. There is a very small difference in the coefficient of thermal expansion between the concrete and steel. The example problem given in 8.4 indicates that these thermal effects are not a significant source of loss.

4.2.4.2 Example of beam with concentric tendon—Consider the simple case of a concentric tendon in a pretensioned beam as shown in Fig. 4.2.4.2. For this discussion, assume that the beam is steam cured overnight, with the steam shut off at some point to allow the beam to cool before the strands are cut.

Assuming the abutments are completely fixed, if the temperature rises $\Delta T_1$ along the entire tendon length while there is no bond between the strand and the concrete, the tension in the tendon will relax by $\Delta T_1 \alpha_m E_p$, where $\alpha_m$ is the coefficient of thermal expansion of the prestressing steel and $E_p$ is the modulus of elasticity.

To simplify the analysis, assume that when the concrete reaches its peak temperature, which is typically maintained for several hours, perfect bond develops between the concrete and steel, and the concrete develops strength and stiffness, with a modulus of elasticity of $E_c$. For this discussion, another simplifying assumption is that the strand segments outside the beam and inside the beam are the same temperature.

After the steam is shut off, the tarp is removed, or forms are stripped, the beam and strand are exposed to ambient temperature, and the steel and concrete will cool before prestress transfer by some amount, $\Delta T_2$. As the concrete and steel cool and try to shorten, the contraction is restrained, so the strand stress becomes more tensile. There will be a change in force, $\Delta P$, which is the same in the free length of strand as it is in the beam. Within the beam, however, a portion of this force is carried by the concrete, $\Delta P_c$, and the remainder is carried by the prestressing steel, $\Delta P_s$. Strain compatibility should be maintained between the concrete and prestressing steel within the beam, so the following Eq. (4.2.4.2a) and (4.2.4.2b) are written.

$$\Delta \varepsilon_p = \Delta \varepsilon_c \tag{4.2.4.2a}$$

$$\Delta T_1 \alpha_m - \frac{\Delta P_s}{E_p A_p} = \Delta T_1 \alpha_s - \frac{\Delta P_s}{E_s A_s} \tag{4.2.4.2b}$$

Equation (4.2.4.2b) is written assuming that the force that develops is tensile. The other relationship that can be written is the total length change of the tendon is zero

$$\Delta \varepsilon_{L\text{,beam}} + \Delta \varepsilon_{L\text{,free}} = 0 \tag{4.2.4.2c}$$

where $\Delta \varepsilon_{L\text{,free}}$ is the strain in the free tendon length, and $L_{free}$ is the total free tendon length. The strain in the free length is calculated as Eq. (4.2.4.2d)

$$\Delta \varepsilon_{L\text{,free}} = \Delta T_1 \alpha_m - \frac{\Delta P_s}{E_p A_p} \tag{4.2.4.2d}$$

When performing example calculations, assuming that the temperature rise and temperature fall are the same, the strand outside the beam will experience a net increase in tension and the concrete will have a net tensile stress as the composite beam resists the restraint forces. Upon release, the restraint forces are removed, and the prestress force at transfer is lower than the original jacking force due to the loss caused by increased temperatures between tensioning and bond on the fixed length bed. Precasters have reported the effects of the tensile stress in the concrete prior to release as causing occasional vertical cracking in prestressed girders before release of the strands (Gross 1998). Typically, these cracks close at prestress transfer.

Based on the prior analysis, concrete has a tensile stress before transfer, and the strand outside the beam has a higher tensile force than the original jacking force.

With these relationships, a rough idea of strand and concrete stresses are established. Many factors, however, complicate the quantification of these stresses. They include the variable nature of early-age concrete modulus of elasticity and the coefficient of thermal expansion of the young concrete. While the cooling occurs over a period of several hours, the modulus and coefficient of thermal expansion are changing and there will be restraint of thermal movements...
by the formwork; both are difficult to quantify. Finally, prior discussion used a concentric tendon. The tendon’s eccentricity will also affect result.

In an analysis, Gross (1998) used a similar method to determine effects of temperature during casting. However, rather than trying to quantify the effect of the developing stiffness of the concrete, a factor was calibrated that accounted for the concrete’s contribution to the stiffness of the embedded strand.

4.2.5 Steel relaxation—Relaxation is a time-dependent property of the prestressing steel, which results in a loss of stress under a constant strain. Relaxation begins immediately after stressing, so it should be considered in the calculation of losses prior to transfer. The following equation, Eq. (4.2.5), is widely accepted for calculating relaxation losses for low-relaxation strands

\[ f_{pe}(t) = f_{ps} \left(1 - \log \left(\frac{f_{ps}}{45} \cdot 0.55\right)\right) \]

where \( t \) is duration of load, in hours; \( f_{ps} \) is initial strand stress after jacking and seating, psi (MPa); and \( f_{ps} \) is yield stress of strand, psi (MPa).

If concrete is cast soon after stressing, and the force is released within 24 hours, the relaxation loss will be relatively small (~1 percent) and can be compensated with an initial overstress. Producers might account for this, but designers would generally not consider this component of the relaxation loss.

4.3—Elastic shortening losses in pretensioned members

Any length change of prestressed reinforcement results in a corresponding change in the effective prestress force. During prestress transfer in a pretensioned member, the prestressing strands shorten, as illustrated in Fig. 4.3, as the surrounding concrete is compressed until equilibrium is achieved. The strain decrease in the strand is accompanied by a corresponding decrease in the strand tension. The resulting change in effective prestress is known as the elastic shortening loss.

Reasonably accurate estimation of elastic shortening prestress loss is important for two reasons:

1. Stresses in the concrete and prestressed reinforcement immediately after transfer should satisfy code-prescribed limits. These limits effectively constrain the member size, prestress amount, and concrete strength at transfer.

2. The instantaneous and time-dependent creep responses of prestressed concrete members are strongly dependent on the initial elastic shortening and state of stress at transfer. Thus, an error in the estimation of the elastic shortening loss can result in magnified errors in predicted camber and long-term prestress losses.

Members with larger concrete precompression stresses, such as bridge girders, usually experience larger elastic shortening losses than more lightly prestressed members. Prestress losses attributable to elastic shortening typically range from 4 to 10 percent of the prestress force before transfer in fully pretensioned members—for example, ACI 318 Class U Class T flexural members. Higher losses are possible in members with lower concrete stiffness, such as those constructed with lightweight concrete. Average elastic shortening losses in post-tensioned members are usually smaller and depend heavily on the sequence of stressing operations.

4.3.1 Transformed-section approach—Computation of stresses in concrete and prestressing steel immediately after transfer is straightforward using the transformed-section approach. Assuming that: a) there is a linearly elastic material behavior; b) there is a perfect bond between concrete and reinforcement; and c) the plane sections remain plane, the following relationships (Eq. (4.3.1a) and (4.3.1b)) result from satisfying axial force and moment equilibrium on a cross section immediately after transfer.

\[ f_{ci} = \frac{f_{ps} A_p}{A_p} - \frac{f_{ps} A_p e_p Y_p}{I_p} + \frac{My_p}{I_p} \]  

(4.3.1a)

and

\[ f_{ps} = f_{ps} - n_p \left(\frac{A_p}{A_p} + e_p^2 \frac{A_p}{I_p}\right) f_{ps} + n_p \frac{Me_p}{I_p} \]  

(4.3.1b)

where \( f_{ps} \) is concrete compressive stress immediately after transfer at fiber under investigation, psi (MPa); \( f_{ps} \) is stress in prestressing steel immediately before transfer, psi (MPa); \( f_{ps} \) is stress in prestressing steel immediately after transfer, psi (MPa); \( A_p \) is area of prestressed reinforcement, in.\(^2\) (mm\(^2\)); \( A_p \) is transformed area of cross section (transforming reinforcement area to concrete area of equivalent stiffness), in.\(^2\) (mm\(^2\)); \( I_p \) is second moment of transformed area of cross section (moment of inertia), in.\(^4\) (mm\(^4\)); \( E_p \) is eccentricity of prestress force with respect to centroid of transformed area, in. (mm); \( Y_p \) is distance from centroid of transformed section to concrete fiber under investigation, in. (mm); \( n_p = E_p/E_{ci} \) is modular ratio of prestressing reinforcement with respect to concrete at transfer; \( E_p \) is modulus of elasticity of prestressed reinforcement, psi (MPa); \( E_{ci} \) is modulus of elasticity of concrete at time of prestress transfer, psi (MPa); and \( M \) is bending moment experienced by cross section immediately after transfer (usually due to self-weight), in.-lb (N-mm).
For continuous members, the total prestress moment (primary plus secondary) replaces $f_{ps}A_{ps}e_p$ in the second term of Eq. (4.3.1a).

Equation (4.3.1a) computes the concrete stress at the level of the prestressing steel immediately after transfer, $f_{ps}$, by setting $y_c$ equal to $e_p$. Equation (4.3.1b) can then be simplified to yield Eq. (4.3.1c)

$$f_{ps} = f_{ph} + n_p f_{c}$$  \hspace{1cm} (4.3.1c)

where $f_{ps}$ is concrete stress at the level of the prestressing steel immediately after transfer, psi (MPa).

Although these equations use the prestress level immediately after transfer, the stresses that result from Eq. (4.3.1a) through (4.3.1c) represent stresses immediately after transfer. Thus, when using the transformed-section approach, the post-transfer concrete stresses are computed directly from the pretransfer prestress force. Accordingly, the prestress loss attributable to elastic shortening, $\Delta \rho_{ES}$, is simply the difference between $f_{ps}$ and $f_{ph}$ as shown in Eq. (4.3.1d) and (4.3.1e).

$$\Delta \rho_{ES} = n_p \left( \frac{A_{ps}}{A_c} + \frac{e_p^2 A_{ps}}{I_c} \right) f_{ps} + n_p \frac{M_c e_p}{I_c} \hspace{1cm} (4.3.1d)$$

or

$$\Delta \rho_{ES} = n_p f_{c} \hspace{1cm} (4.3.1e)$$

The first two terms in Eq. (4.3.1d) represent the effect of strand shortening as the concrete is compressed; the third term represents the opposing effect of the self-weight moment on the cross section.

4.3.2 Gross-section approximation—If computation of transformed-section properties is considered overly burdensome, the elastic shortening loss can be estimated with reasonable accuracy using gross-section properties. This is specifically true for lightly prestressed members. For linearly elastic material behavior and perfect bond between reinforcement and concrete, the change in prestress during transfer is computed by multiplying concrete stress at the level of the prestressing steel after transfer ($f_{cor}$) by the modular ratio ($n_p$), as indicated in Eq. (4.3.1e).

The concrete stress $f_{cor}$ is computed using gross-section properties, but prior knowledge of the effective prestress force after transfer is required. The PCI Design Handbook (PCI 2010) simplifies the method using an approximate version of this technique. A single iteration of the calculation in Eq. (4.3.1e) is performed after an initial assumption of a 10 percent prestress loss due to elastic shortening. This assumption is embodied in the handbook's value of $K_{cor} = 0.9$ for pretensioned members. Equations (4.3.2a) and (4.3.2b) are used to calculate elastic shortening loss

$$\Delta \rho_{ES} = \frac{E_p}{E_c} f_{cor} \hspace{1cm} (4.3.2a)$$

$$f_{cor} = K_{cor} \left( \frac{P}{A_c} + \frac{P e_p}{I_c} \right) - \frac{M_c e_p}{I_c} \hspace{1cm} (4.3.2b)$$

where $P_i$ is initial prestress force (after anchorage seating loss), lb (N); $e_p$ is eccentricity of prestressed reinforcement with respect to the gross section, in. (mm); $A_c$ is area of the gross cross section, in.$^2$ (mm$^2$); $I_c$ is second moment of area of gross cross section (moment of inertia), in.$^4$ (mm$^4$); and $M_c$ is bending moment due to dead weight of prestressed member and any other permanent loads in place at the time of stressing, in.-lb (N-mm).

4.3.3 Iterative gross-section approach—A more exact value of the elastic shortening loss is obtained using gross-section properties when an iterative solution technique is employed. A trial value of elastic shortening loss is assumed and used to compute $f_{cor}$. After Eq. (4.3.1e) is used to update elastic shortening loss, the designer can evaluate the accuracy of the initial estimate and reiterate if desired. Two or three iterations are usually adequate to get a reasonable level of accuracy. Alternatively, a closed-form solution for directly computing the elastic shortening loss with gross-section properties is employed to condense repeated iterations to a single step.

$$\Delta \rho_{ES} = n_p M_c e_p A_e \left( \frac{f_{ps}}{A_c} + \frac{e_p^2 A_{ps}}{I_c} \right) - \frac{A_{ps}}{n_p} \left( \frac{f_{ps}}{A_c} + \frac{e_p^2 A_{ps}}{I_c} \right)$$

$$+ \frac{M_c e_p}{I_c} \hspace{1cm} (4.3.3)$$

Equation (4.3.3) is functionally equivalent to the expression recommended in the AASHTO LRFD (2011) commentary.

Analysis with gross-section properties results in an elastic shortening approximation that is close to the transformed-section value if: a) the closed-form solution of Eq. (4.3.3) is employed; or b) an iterative approach is pursued until convergence is achieved. If the iterative approach is used with net section properties, the solution will nearly match the transformed section properties solution. However, accuracy of the single-iteration approximation of the PCI Design Handbook (PCI 2010) depends on how closely the assumed elastic shortening loss of 10 percent matches the final calculated value.

The example pretensioned double-tee beam in 8.1 provides a basis for comparison between the methods. Transformed-section analysis (8.1.2) results in a computed elastic shortening loss of 15,700 psi (108 MPa), which is approximately 7.8 percent of the pretransfer stress. Application of Eq. (4.3.3) using gross-section properties yields a computed loss of 15,550 psi (108 MPa) (7.7 percent). The single-iteration approximation from the PCI Design Handbook (PCI 2010) simplified method using $K_{cor} = 0.9$ is 14,900 psi (103 MPa) (7.4 percent). For this example, the relative error among these three computational techniques is less than 0.5 percent of the initial prestress level, which is small compared to the uncertainty associated with the actual material properties. In general, the small relative error between transformed-section (and iterative with net section) and gross-section
analyses will increase as the ratio of bonded reinforcement to member cross-sectional area increases.  

4.3.4 Variation of elastic shortening loss along the member length—The bending moment due to self-weight varies along the member length. Therefore, the magnitude of the elastic shortening loss is not uniform along the pretensioned member. Variation in the eccentricity or quantity of the bonded prestressed reinforcement is often employed to control concrete stresses after transfer. ACI 318 requires that member sections satisfy the prescribed stress limits. It is usually adequate to compute elastic shortening losses at cross sections that are likely to be critical for concrete stress checks immediately after transfer. If little variation in the effective prestress force is expected along the member, it is common practice to use the elastic shortening loss computed for the midspan cross section. In members with single-point strand draping and a parabolic moment diagram from gravity loads, the cross section located at 40 percent of the simple span is often selected. End regions are often critical for concrete stress checks immediately after transfer.

4.3.5 Sensitivity to concrete stiffness—Inspection of Eq. (4.3.1d) indicates that the accuracy of computed elastic shortening loss is dependent on accurate estimation of the concrete modulus of elasticity at prestress transfer \(E_{ci}\). The elastic shortening loss depends directly on concrete material stiffness, a property that is also one of the most uncertain of the parameters involved in computing this loss. As discussed in 7.4, actual measured \(E_{ci}\) values can vary greatly from values predicted in accordance with simple, code-prescribed equations; therefore, codes often allow the use of mixture-specific stiffness values if they are known. Knowledge of the stiffness characteristics of commonly used aggregates and concrete mixtures in a region are used to improve the accuracy of the \(E_{ci}\) value assumed for design. Even when the relationship between concrete compressive strength and \(E_{ci}\) is well established, variability is often experienced because measured concrete compressive strengths usually exceed specified values to reduce the time that the prestressed element remains in the casting bed.

4.4—Post-tensioning losses during tensioning and transfer

4.4.1 Initial losses and tendon types—The initial losses in post-tensioning (PT) tendons are friction, anchor set, and elastic shortening. Although there are many types of PT tendons, only the most commonly used ones are discussed herein. The stressing sequence affects the initial losses of all tendons.

Unbonded single-strand tendons are coated with PT coating and sheathed with extruded high-density polyethylene (HDPE) sheathing. The PT coating lubricates the strand that can move freely within the sheathing, which reduces friction. These tendons are stressed with a single-strand jack and the elongations measured after anchor set and after the stressing jack has been removed from the tendon.

Multistrand tendons typically consist of multiple bare strands in a metal or plastic duct. The strands are stressed together with a center-hole multistrand jack; however, the elongations are measured before anchor set. Multistrand tendons are normally grouted with cementitious grout after stressing and elongation verification, and are often called bonded tendons.

Bar tendons typically consist of a single bar in a metal or plastic duct. The bar is stressed with a bar jack using a coupler and a pulling bar. The elongation is typically measured after anchor set, or the jacking force is verified with a lift-off test. Bar tendons are grouted after stressing.

4.4.2 Friction losses—During stressing, the prestressing force along a tendon decreases from its maximum value at the stressing end to a minimum at the fixed end. This decrease is due to friction losses between the prestressing strand and its sheathing or duct due to the intended tendon curvature and unintended tendon curvature, which is also called wobble. Once the jacking force reaches its prescribed value, the tendon is released and anchored at the stressing end. Locking the tendon is achieved in most cases by the engagement of serrated wedges, which are activated as the strand retracts and draws the wedges into a conical wedge cavity in the anchor or a wedge plate. Therefore, during stressing, there are two components of initial stress loss: one from friction and the other from anchor set. An example of a final stress diagram of a tendon with one stressing end immediately after anchor set is shown in Fig. 4.4.2a. The slope of the curve, \(-s\), between \(f_{jock}\) and \(f_{ic}\) represents the average friction intensity. The triangular area between \(f_{jock}\), \(f_{anchors}\), and \(f_{max}\) divided by \(E_p\) represents the loss of elongation due to the anchor set. The curve between \(f_{anchors}\), \(f_{max}\), and \(f_{ic}\) represents the strand stress along the length after anchor set. The area under this curve divided by \(E_p\) is the net tendon elongation after anchor set.

The difference in strain stress between the stressing end and the fixed end increases with increasing tendon length, greater friction coefficient, and larger angular change in the tendon profile.

In typical building construction with unbonded single-strand tendons, the slope of the stress curve will be relatively flat as the friction is small. For normal-length tendons, the average tendon stress is assumed to act at all critical sections along the length. For long tendons, over 120 ft (37 m) in length when stressed from one end only, or over 240 ft (73 m) in length when stressed from both sides, additional tendons may be required to provide the total prestressing force required at a given location in the member. For short tendons less than 40 ft (12 m) in length, the effect of the anchor set can be significant and should be considered as discussed in 4.4.3.

The stress distribution for a tendon stressed from both ends is illustrated in Fig. 4.4.2b. It is not necessary to stress the tendon at both ends simultaneously. The stressing at the second end typically takes place after stressing of the first end is completed.

As shown in Fig. 4.4.2b, when the resulting stress distribution is symmetrical when the tendon is stressed from both ends, the average friction is also symmetrical. Figure 4.4.2b also illustrates that the elongation before anchor set obtained from the first-end stressing, proportional to the
Stressing

Fig. 4.4.2a—Stress distribution in tendons with one stressing end.

First Stressing

Second Stressing

area under the stress curve, $f_{\text{jack}}$ and $f_1$, will be much larger than the remaining elongation obtained from the second-end stressing, proportional to the area between $f_{\text{jack}}$, $f_{\text{jack}}$, and $f_1$.

Bonded multistrand tendons are usually used in bridges with deeper members, resulting in a greater angular deviation. They are also used in buildings, in transfer girders, in beams, in slabs, and also in other structures such as water tanks. As the friction increases over the supports where the angular deviation is larger, each tendon is analyzed separately so that the actual force in each span and at each critical section is evaluated. Figure 4.4.2c shows the stress diagram with variable friction intensity. In bonded tendons, no force equalization occurs, as the tendons are typically grouted shortly after stressing and an average force for the tendons is typically not used.

Friction losses are subdivided into intended tendon curvature, represented by a curvature friction coefficient $\mu$, and the unintended tendon curvature, represented by a wobble friction coefficient $k$. The stiffness of the duct contributes to a lower wobble friction, and a more flexible duct could contribute to increased wobble friction. The typically stiffer metal ducts have a larger maximum support spacing requirement than the more flexible plastic ducts. The effect of friction on the force along the tendon is calculated according to the formula

$$ T_x = T_e e^{-\mu x} \Delta s$$

(4.4.2)

Friction coefficients are specific to tendon type and configuration. For unbonded single-strand tendons, the type of PT coating and characteristics of the extruded plastic sheathing determine the friction values. As the PT coating and extruded plastic sheathing materials improve in quality and consistency, predictability of the friction values increases in accuracy. For bonded multistrand tendons, the type of duct is of great importance. For example, plastic ducts typically have lower friction values than corrugated metal ducts. Recommended friction coefficients are shown in Table 4.4.2 from the Post-Tensioning Institute’s guide specifications (PTI M50.3-12 Table 5.1).

Bar tendons are typically straight and relatively short with no friction along the tendon length as long as the bar is not touching the duct.

4.4.3 Anchor set—A typical value for the anchor set is 1/4 in. (6 mm) when stressing equipment with a power seating device is used. When single-strand tendons are stressed with jacks without a power seating device, anchor setup to 3/8 in. (19 mm) is common.

The effect of anchor set on tendon stresses is calculated in accordance with the formulas that follow. The anchor set is computed as the integration of the stress from the jacking point to the anchor set influence distance divided by the modulus of elasticity.

$$ \Delta s = \frac{\int_{0}^{L} (\text{final stress} - \text{initial stress})dx}{E_p} \quad (4.4.3a)$$
### Table 4.4.2—Recommended friction coefficients

<table>
<thead>
<tr>
<th>Type of prestressing steel</th>
<th>Corrugated metal duct</th>
<th>Corrugated plastic duct</th>
<th>Smooth steel pipe</th>
<th>Smooth plastic pipe</th>
<th>No duct plastic sheathing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$, k ft $^{-1}$ (m$^{-1}$)</td>
<td>$\mu$, k ft$^{-1}$ (m$^{-1}$)</td>
<td>$\mu$, k ft$^{-1}$ (m$^{-1}$)</td>
<td>$\mu$, k ft$^{-1}$ (m$^{-1}$)</td>
<td>$\mu$, k ft$^{-1}$ (m$^{-1}$)</td>
</tr>
<tr>
<td>Strand</td>
<td>0.15 to 0.25</td>
<td>0.00005 to 0.0003 (0.0002 to 0.0010)</td>
<td>0.10 to 0.14</td>
<td>0.00005 to 0.0003 (0.0002 to 0.0010)</td>
<td>0.25 to 0.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strand in precast elements and constant curvature tendons</td>
<td>0.15 to 0.25</td>
<td>0.00005 to 0.0003 (0.0002 to 0.0010)</td>
<td>0.10 to 0.14</td>
<td>0.00005 to 0.0003 (0.0002 to 0.0010)</td>
<td>0.25 to 0.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>External tendons, bare dry strand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lubricated strand</td>
<td>0.12 to 0.18</td>
<td>0.00005 to 0.0003 (0.0002 to 0.0010)</td>
<td>0.20 to 0.25</td>
<td>0.00005 to 0.0003 (0.0002 to 0.0010)</td>
<td>0.20 to 0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strand coated and extruded</td>
<td>0.01 to 0.05</td>
<td>0.00005 to 0.0003 (0.0002 to 0.0010)</td>
<td>0.01 to 0.05</td>
<td>0.00005 to 0.0003 (0.0002 to 0.0010)</td>
<td>0.01 to 0.05</td>
</tr>
<tr>
<td>Bars, deformed, smooth, and round</td>
<td>0.30</td>
<td>0 to 0.0002 (0 to 0.0007)</td>
<td>0.30 to 0.0002 (0 to 0.0007)</td>
<td>0.30 to 0.0002 (0 to 0.0007)</td>
<td>0.30 to 0.0002 (0 to 0.0007)</td>
</tr>
</tbody>
</table>

*Post-tensioned coating in accordance with the performance specification (PTI M10.3-00).*

\[
\Delta s = \frac{\Delta f_{pt} x_s}{2 E_p} \quad (4.4.3b) \\
\Delta f_{pt} = 2 \sqrt{\frac{E_p d \Delta s}{L}} \quad (4.4.3g)
\]

The slope of the stress line represents the friction intensity or the friction over the length. The slope of the stress line after anchor set is equal and opposite to the slope of the jacking stress line. Equation (4.4.3c) only applies when the friction is linear between the two ends (or points under consideration).

\[
s = \frac{d}{L} = \frac{f_{jack} - f_s}{L} \quad (4.4.3c)
\]

\[
x_s = \frac{\Delta f_{pt}}{2s} = \frac{\Delta f_{pt} L}{2d} \quad (4.4.3d)
\]

\[
\Delta f_{pt} = \frac{2 d x_s}{L} \quad (4.4.3e)
\]

Substituting Eq. (4.4.3e) into Eq. (4.4.3b), and solving for $x_s$ gives

\[
x_s = \sqrt{\frac{E_p \Delta s L}{d}} \quad (4.4.3f)
\]

If $x > L$, then the loss transfers all the way to the fixed end, as shown in Fig. 4.4.3.

Or, Eq. (4.4.3e) can be substituted into Eq. (4.4.3b) to arrive at Eq. (4.4.3g) for $\Delta f_{pt}$

\[
\Delta = \frac{P_{avg} L}{A_p E_p} \quad (4.4.4.1)
\]
Fig. 4.4.3—Stress distribution in short tendons.

where Δ is elongation, in. (mm); \( P_{avg} \) is average force in the tendon, lb (N); \( E \) is the average stress in the tendon, psi (MPa); \( L \) is strand length from anchorage to anchorage, in. (mm); \( A_p \) is area of prestressing steel, in.² (mm²); and \( E \) is modulus of elasticity of prestressed reinforcement, psi (MPa).

The average prestressing force depends on friction losses caused by tendon curvature, intended as well as wobble, and the anchor set.

Values for strand area and modulus of elasticity are verified based on mill certificates of the steel used for the particular tendon if elongation variations are greater than permissible.

4.4.4.2 Elongation measurements—The procedures for tendon stressing and field measurements of elongations are in industry standard documents (PTI M10.3-00; PTI C30.4-07).

4.4.4.3 Items considered for elongations—In cases of elongations outside of the specified tolerances, several factors should be verified or considered:

(a) Problems, inconsistencies, or both, in tendon marking and measurement of the elongation

(b) Excessive friction due to greater angular deviation, larger friction, or both, than assumed; or excessive wobble caused by placement variations

(c) Excessive anchor set caused by cement paste in the wedge cavity or improperly functioning stressing equipment or power seating device

(d) Variations in steel area, modulus of elasticity, or both, that can be verified on mill certificates for strand supplied to the job

(e) Error in calculations; shorter tendons are affected more than longer ones by inaccuracy in measurements and in a variation in anchor set as these values represent a larger percentage of the overall elongation. The anchor set in short tendons affects the tendon stress to the fixed end.

4.4.4.4 Elongations—The jacking force is the primary concern when stressing tendons. Calibrated stressing jacks and gauges should be used while the elongation measurements serve as a verification of the jacking force. Because of the many factors affecting elongations, a reasonable tolerance between the jacking force and the measured elongation should be used.

Different codes require specific tolerances for the correlation between the expected and measured elongations.

Table 4.4.4.4—Tolerances for the correlation between the expected and measured elongations in selected codes

<table>
<thead>
<tr>
<th>Code</th>
<th>Tolerance in elongation expected versus measured, percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI 318-11</td>
<td>± 7 percent</td>
</tr>
<tr>
<td>AASHTO (1989)</td>
<td>± 5 percent on individual tendon based on friction coefficients confirmed with liftoff tests, and material properties of the actual materials used</td>
</tr>
<tr>
<td>AASHTO (2011)</td>
<td>± 5 percent for tendon length over 50 ft (15 m)</td>
</tr>
<tr>
<td></td>
<td>± 7 percent for tendon length 50 ft (15 m) or less</td>
</tr>
<tr>
<td>Eurocode EN 13670:2009</td>
<td>± 5 percent for the total force at a section</td>
</tr>
<tr>
<td></td>
<td>± 15 percent for an individual tendon</td>
</tr>
<tr>
<td>CEB-FIP model code (Comité Euro-International du Béton 2010)</td>
<td>Tendon L &gt; 50 ft (15 m): ±10 percent for a particular tendon</td>
</tr>
<tr>
<td></td>
<td>±5 percent for all tendons in a section</td>
</tr>
<tr>
<td></td>
<td>Tendon L ≤ 50 ft (15 m): ±15 percent for a particular tendon</td>
</tr>
<tr>
<td></td>
<td>±7 percent for all tendons in a section</td>
</tr>
</tbody>
</table>

Table 4.4.4.4 summarizes such tolerances. These tolerance approaches are used as guidance in resolving elongation readings outside the specified tolerances.

As indicated in Table 4.4.4.4, there are different tolerance percentages in codes for:

(a) Long and short tendons
(b) Particular tendons and all tendons in a section or a member
(c) Under elongation and over elongation

4.5—Elastic shortening loss in post-tensioned members

In post-tensioned members, a stressed tendon is elongated at the same time the surrounding concrete is being compressed. Thus, this tendon experiences no elastic shortening during its own stressing operation. Previously anchored tendons, however, shorten with the surrounding concrete during the subsequent stressing of other tendons and, therefore, experience prestress loss due to elastic shortening. The elastic shortening loss in a post-tensioned member depends on the sequence of stressing operations and is usually taken as an average of the losses experienced by the individual tendons.

At the time of stressing, all post-tensioned tendons are unbonded. Because an unbonded tendon can move within its sheathing or duct, it does not undergo the same stress-induced strain changes as the concrete surrounding it for most flexural members. For this reason, the average compressive stress in the concrete is suggested for use in evaluating prestress losses due to elastic shortening. This procedure relates the elastic shortening prestress loss for unbonded tendons to the average strain along the length of the member rather than the strain at the point of maximum moment.

In Zia et al. (1979), elastic shortening losses are calculated as

\[
\Delta f_{eks} = K_{ps}E_p f_{psd}/E_{ci} \tag{4.5.1}
\]
where $f_{con}$ is average compressive concrete stress at the center of gravity of the tendons immediately after the prestress has been applied to the concrete (considerations for computing this average stress are discussed later in this section), psi (MPa). $K_{et} = 0.5$ for post-tensioned components when tendons are tensioned in sequential order to the same tension and 0 if all tendons are tensioned simultaneously; with other PT procedures, the value for $K_{et}$ may vary from 0 to 0.5. $E_p$ is modulus of elasticity of prestressed reinforcement (approximately 28,500,000 psi [196,500 MPa]).

Computation of elastic shortening losses in post-tensioned members can differ relative to similar computation for pretensioned members. As discussed previously, the elastic shortening loss is estimated as the average loss experienced by all tendons due to all stressing operations. This is accomplished by using a $K_{et}$ value of 0.5 as an averaging factor for post-tensioned members when tendons are tensioned in sequential order to the same tension (Zia et al. 1979). Similarly, the AASHTO (2011) specification recommends an averaging factor of $(N - 1)/2N$, where $N$ is the number of sequentially stressed tendons. In cases where large tendon spacing results in minimal elastic interaction between adjacent tendons, such as some slab systems, the elastic shortening losses in post-tensioned tendons may be further reduced. AASHTO (2011) recommends a further 75 percent reduction of the computed elastic shortening loss in this case; the Zia et al. (1979) simplified method leaves the selection of a reduced $K_{et}$ value somewhere between 0 and 0.5, which is based on the discretion of the practitioner.

When averaging the elastic shortening losses among sequentially stressed tendons, the influence of self-weight bending moment should be considered. For the case of a post-tensioned beam, the self-weight moments are likely to be induced simultaneously with the first tendon stressing operation that lifts the beam from its supports; therefore, they are already compensated for when stressing of this tendon is completed. In this case, the elastic shortening losses that occur due to subsequent stressing operations will be due to prestress forces only.

A second major difference related to computation of elastic shortening effects in post-tensioned members is that the tendons are unbonded at the time of prestress transfer. For this reason, net-section properties are used in the computation of elastic shortening loss and the accompanying concrete stresses after transfer. Net-section properties are calculated by deducting the cross-sectional area of the PT ducts. If a large amount of bonded reinforcement—prestressed or nonprestressed—is included, transformed-section analysis using net-section concrete properties can improve accuracy of elastic computations. This involves transforming the area of bonded reinforcement into an equivalently stiff area of concrete, and deducting the area of open ducts.

Also, because the tendons are unbonded at prestress transfer, shortening of the tendon is not directly related to the concrete strain at the cross section of interest. The elastic shortening of a tendon is a direct function of the relative movement of its two anchors. Thus, for a tendon with concentric anchors, the elastic shortening deformation can be computed directly from the average prestress on the cross section—as is done in the post-tensioned slab example in 8.2. Note that for draped or nonconcentric straight tendons, the average concrete stress along the full length of the tendon at its center of gravity has to be calculated to determine the elastic shortening loss.

Finally, as discussed in 4.4, friction losses result in a prestress force that varies along the length of a post-tensioned member. Therefore, the average prestress force over the length of the member after friction losses can be used when estimating elastic shortening losses in these members.

### 4.6—Elastic gain under superimposed loads

When superimposed dead loads are added at a later age, the prestress reinforcement can elongate and cause an increase in effective prestress force. This increase is effectively a partial recovery of prestress loss that is sometimes called elastic gain. Superimposed dead loads that may result in elastic gain include cast-in-place toppings on flooring members and decks and traffic barriers on bridge girders. For pretensioned members and post-tensioned members with bonded tendons, the elastic gain is calculated by multiplying the resulting change in concrete stress at the center of gravity of the prestress force by the modular ratio

$$\Delta p_{EG} = n_p \Delta f_{egp}$$  (4.6)

where $\Delta f_{egp}$ is increase in prestress (elastic gain) due to addition of superimposed dead load, psi (MPa); $n_p = E_p/E_c$ = modular ratio of prestressing reinforcement with respect to concrete at age when superimposed load is applied; and $\Delta f_{egp}$ is change in concrete stress at center of gravity of the prestressing force due to application of superimposed load, psi (MPa).

Little to no increase in effective prestress force is expected to accompany the addition of superimposed dead loads to PT members with unbonded tendons because the tendon length will change only slightly between anchorages.

### 5.0—Long-term losses: simplified method

#### 5.1—Scope

Long-term prestress losses are caused by the time-dependent properties of concrete and steel, namely, concrete creep and shrinkage and steel relaxation. Determination of long-term prestress loss can involve complicated and laborious procedures because the rate of loss due to one factor, such as relaxation of tendons, is continually altered by changes in stress due to other factors, such as shrinkage and creep of concrete. Rate of creep is, in turn, altered by the change in tendon stress. Many of these factors are further dependent on uncertain material properties, loading time, concrete curing method, environmental conditions, and construction details.

The equations presented within this chapter are intended to reasonably estimate prestress loss from the various time-dependent sources. They are applicable for prestressed members of normal designs with: a) an extreme fiber...
5.2—Creep of concrete (Δf_{icorn})

Creep of concrete complicates stress-loss calculations. The rate of loss due to creep changes when the concrete stress level changes. Because the stress level is changing constantly throughout the life of the structure, the rate of loss due to creep is also constantly changing.

Part of the initial compressive strain induced in the concrete immediately after transfer is reduced by the tensile strain resulting from the superimposed permanent dead load. Prestress loss due to concrete creep can be assumed to be proportional to the net permanent compressive strain in the concrete at the level of the reinforcement.

For prestressed members made of sand lightweight concrete, there is a significantly larger amount of loss due to elastic shortening of concrete because of its lower modulus of elasticity. This results in an overall reduction in loss due to creep. This effect is accounted for by a 20 percent reduction of the creep coefficient. For members made of all lightweight concrete, special consideration should be given to the properties of the particular lightweight aggregate used.

### 5.2.1 Bonded tendons—Losses due to creep of concrete are calculated as (Zia et al. 1979)

\[ \Delta f_{icorn} = K_{cr}(E_p/E_c)(f_{cr} - f_{crs}) \]  

(5.2.1a)

where \( K_{cr} \) is 2.0 for pretensioned normalweight components, 1.6 for pretensioned sand-lightweight components, 1.6 for post-tensioned normalweight components, 1.28 for post-tensioned sand-lightweight components; \( f_{cr} \) is net compressive concrete stress at center of gravity of prestressing force immediately after the prestress has been applied to the concrete, psi (MPa); and \( f_{crs} \) is concrete stress at center of gravity of prestressing force due to all superimposed, permanent dead loads that are applied to the member after it has been prestressed, psi (MPa)

\[ f_{crs} = \frac{M_{sd} - e_p}{I_p} \]  

(5.2.1b)

\( E_c \) is modulus of elasticity of concrete at 28 days, psi (MPa); \( M_{sd} \) is moment due to all superimposed, permanent dead, and sustained loads applied after prestressing, lb-in. (N-mm); and \( e_p \) is eccentricity of the center of gravity of the tendons with respect to the centroid of the concrete at the cross section considered, in. (mm).

5.2.2 Unbonded tendons—Because an unbonded tendon can move within its sheathing, it does not undergo the same stress-induced strain changes as the concrete surrounding it. For this reason, the average compressive concrete stress at the center of gravity of the tendon, \( f_{crs} \), is suggested for use in evaluating prestress losses due to creep of concrete. This procedure relates the prestress loss for unbonded tendons due to creep to the average member strain at the center of gravity of the tendon, rather than the strain at the center of gravity of the tendon at the section of maximum moment. The somewhat higher residual tensile stress in an unbonded tendon logically results in somewhat higher loss due to steel relaxation.

\[ \Delta f_{icorn} = K_{cr}(E_p/E_c)f_{crs} \]  

(5.2.2)

where \( f_{crs} \) is average compressive concrete stress along the member length at the center of gravity of the tendons immediately after the prestress has been applied to the concrete, psi (MPa).

5.3—Concrete shrinkage (Δf_{sh})

Loss of stress in the tendon due to concrete shrinkage surrounding it is proportional to that part of the shrinkage that takes place after the transfer of prestress force to the concrete. Shrinkage strain developed in a concrete member is influenced, among other factors, by its volume/surface ratio and the ambient relative humidity. Prestress loss due to shrinkage is the product of the shrinkage that occurs after transfer and the modulus of elasticity of prestressing steel. The factor \( K_{sh} \) accounts for the reduction in shrinkage due to delaying the application of the prestressing force.

Note that for some lightweight concrete, the coefficient for the estimation of shrinkage strain may be greater than the value \( 8.2 \times 10^{-6} \) used in Eq. (5.3).

Losses due to shrinkage of concrete are calculated:

\[ \Delta f_{sh} = 8.2 \times 10^{-6} K_{sh} E_p (l = 0.066/\sqrt{S}(100 - RH)) \]  

(5.3)

\( f_{sh} \) = net compressive concrete stress at center of gravity of prestressing force immediately after the prestress has been applied to the concrete, psi (MPa) (Eq. 4.3.2b); and \( f_{crs} \) is concrete stress at center of gravity of prestressing force due to all superimposed, permanent dead loads that are applied to the member after it has been prestressed, psi (MPa)
Table 5.3—Values of $K_{sh}$ for post-tensioned members (PCI 2010)

<table>
<thead>
<tr>
<th>Time after end of moist curing to application of prestress, days</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{sh}$</td>
<td>0.92</td>
<td>0.85</td>
<td>0.80</td>
<td>0.77</td>
<td>0.73</td>
<td>0.64</td>
<td>0.58</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Fig. 5.3—Annual average ambient relative humidity, percent (PCI 2010).

where $K_{sh} = 1.0$ for pretensioned components—refer to Table 5.3 for post-tensioned components; $V/S$ is volume-to-surface ratio, in. (mm); and $RH$ is average ambient relative humidity, percent (Fig. 5.3).

Except for the $K_{sh}$ term, there is no distinction between pretensioned and post-tensioned members or between those with bonded and unbonded tendons for shrinkage loss calculations.

5.4—Relaxation of tendons ($\Delta f_{pre}$)

Relaxation of tendons complicates stress-loss calculations. The rate of loss due to relaxation changes as the steel stress level changes. Because the stress level is changing constantly throughout the life of the structure, particularly in pretensioned members, the rate of loss due to relaxation is also constantly changing.

Relaxation of a prestressing tendon depends on the stress level in the tendon. Basic relaxation values $K_{re}$ for prestressing steel are shown in Table 5.4. Because of other prestress losses, there is a continual reduction of the long-term tendon stress, thus causing a reduction in the rate of relaxation over time. The reduction in tendon stress due to elastic shortening of concrete occurs instantaneously, while the reduction due to creep and shrinkage takes place over a prolonged period of time. The factor $J$ is specified to approximate these effects.

Losses due to relaxation of tendons are calculated in Eq. (5.4)

$$\Delta f_{pre} = [K_{re} - J(\Delta f_{pSH} + \Delta f_{pCR} + \Delta f_{pED})]C$$  \hspace{1cm} (5.4)

Table 5.4—Values of $K_{re}$ and $J$ (PCI 2010)

<table>
<thead>
<tr>
<th>Tendon type</th>
<th>$K_{re}$, psi (MPa)</th>
<th>$J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 270, stress-relieved strand or wire</td>
<td>20,000 (138)</td>
<td>0.150</td>
</tr>
<tr>
<td>Grade 250, stress-relieved strand or wire</td>
<td>18,500 (128)</td>
<td>0.140</td>
</tr>
<tr>
<td>Grade 240 or 235, stress-relieved wire</td>
<td>17,600 (121)</td>
<td>0.130</td>
</tr>
<tr>
<td>Grade 270, low-relaxation strand</td>
<td>5000 (34)</td>
<td>0.040</td>
</tr>
<tr>
<td>Grade 250, low-relaxation wire</td>
<td>4630 (32)</td>
<td>0.037</td>
</tr>
<tr>
<td>Grade 240 or 235, low-relaxation wire</td>
<td>4400 (30)</td>
<td>0.035</td>
</tr>
<tr>
<td>Grade 145 or 160, stress-relieved bar</td>
<td>6000 (41)</td>
<td>0.050</td>
</tr>
</tbody>
</table>

where values for $K_{re}$ and $J$ are taken from Table 5.4.

The following sections provide equations for the calculation of the coefficient $C$.

5.4.1 Stress-relieved strand—Historically, stress-relieved strands were widely employed in the United States and elsewhere into the mid-1980s. Since then, most of the prestressing strands manufactured and used throughout the world qualify as low-relaxation strands. Stress-relieved strand, also known as normal-relaxation strand, is not currently manufactured in the United States or Canada. Because some historical or older structures may contain stress-relieved strands and it is possible that stress-relieved strand be supplied by manufacturers outside of North America, related equations are presented. If normal-relaxation strand is ordered, manufacturers typically supply low-relaxation strand, which meets all of the requirements of normal-relaxation strand.

In the unlikely case that stress-relieved strands are specified by the licensed design professional, the relaxation properties of the strand supplied will, most likely, more closely conform to those of low-relaxation strand. The installation of strand that behaves similarly to low-relaxation strand, when normal-relaxation strand properties were used for design, results in the oversupply of prestressing force. This could cause increased creep and excessive camber.

Therefore, unless the strand is specified and supplied as low-relaxation, the relaxation properties are measured independently. The equations for stress-relieved strand that are published in this guide and elsewhere (PCI 2010) are not recommended for current applications.

For stress-relieved strand, the coefficient $C$ in Eq. (5.4) is computed as follows.

If $0.75 \geq f_{p0}/f_{pum} \geq 0.70$, $C = 1 + 9(f_{p0}/f_{pum} - 0.7)$ \hspace{1cm} (5.4.1a)

If $0.70 > f_{p0}/f_{pum} \geq 0.51$, $C = [(f_{p0}/f_{pum})0.19][(f_{p0}/f_{pum})0.85 - 0.55]$ \hspace{1cm} (5.4.1b)

If $f_{p0}/f_{pum} < 0.51$, $C = (f_{p0}/f_{pum})/3.83$ \hspace{1cm} (5.4.1c)

5.4.2 Low-relaxation strand—The coefficient $C$ in Eq. (5.4) for low-relaxation strand is computed as follows.

If $f_{p0}/f_{pum} \geq 0.54$, $C = [(f_{p0}/f_{pum})0.21][(f_{p0}/f_{pum})0.9 - 0.55]$ \hspace{1cm} (5.4.2a)

If $f_{p0}/f_{pum} < 0.54$, $C = (f_{p0}/f_{pum})/4.25$ \hspace{1cm} (5.4.2b)
where \( f_{pu} = P/A_{psu} \), psi (MPa); and \( f_{pu} \) is ultimate strength of prestressing steel, psi (MPa).

5.5—AASHTO LRFD approximate estimate of time-dependent losses

A simple equation to estimate the time-dependent prestress loss due to creep of concrete, shrinkage of concrete, and relaxation of steel in typical bridge beams is presented in the “AASHTO LRFD Bridge Design Specifications” (AASHTO 2012). The following criteria should be met to use the equation:

(a) Members are made of normal-weight concrete
(b) The concrete is either steam- or moist-cured
(c) Prestressing is provided by strands or bars with either stress-relieved or low-relaxation properties
(d) Average exposure conditions and temperatures characterize the site

Section C5.9.5.3 commentary of AASHTO (2012) notes that the equation was derived for sections with composite decks and bonded tendons. AASHTO does not define average conditions. In the derivation (Tadros et al. 2003), it was assumed that at the bottom fibers of a composite bridge beam, the individual stresses due to effective prestress, girder weight, deck weight, superimposed dead and live load sum to zero at time infinity. The concrete stress due to external loads at the center of gravity of the strands was assumed to be approximately equally caused by girder weight, deck weight, and live load. Further assumptions stated by Tadros et al. (2003) include:

(a) Cross section is subjected to positive bending moment (composite deck in compression)
(b) Volume/surface ratio of precast section is between 3 and 4 in. (76.2 and 101.6 mm)
(c) No nonprestressed reinforcement is present in the cross section
(d) Prestress is transferred at 1 day of concrete age under accelerated plant curing conditions
(e) Deck weight is applied to unshored precast section at least 28 days after prestress transfer

With these assumptions, the detailed method (6.3.2) is simplified to

\[
\Delta f_{re, T} = 10.0 \frac{f_{ps}}{A_g} \gamma_h \gamma_{st} + 12,000 \gamma_h \gamma_{st} + \Delta f_{re, E}
\]

(for stresses in psi)

\[
\Delta f_{re, T} = 10.0 \frac{f_{ps}}{A_g} \gamma_h \gamma_{st} + 83 \gamma_h \gamma_{st} + \Delta f_{re, E}
\]

(for stresses in MPa)

where

\[
\gamma_h = 1.7 - 0.01 RH
\]

\[
\gamma_{st} = \frac{5000}{1000 + f_{ps}} \quad \text{(for } f_{ps} \text{ in psi)}
\]

\[
\gamma_{st} = \frac{34.5}{6.9 + f_{ps}} \quad \text{(for } f_{ps} \text{ in MPa)}
\]

where \( f_{ps} \) is prestressing steel stress immediately following transfer, psi (MPa); \( A_{psu} \) is area of prestressing steel, in.\(^2\) (mm\(^2\)); \( A_g \) is gross cross-sectional area of beam, in.\(^2\) (mm\(^2\)); \( \Delta f_{re, E} \) is estimate of relaxation loss = 2400 psi (16.5 MPa) for low-relaxation strand and 10,000 psi (68.95 MPa) for stress-relieved strand; \( \gamma_h \) is correction factor for ambient relative humidity; \( \gamma_{st} \) is correction factor for specified concrete compressive strength at transfer; \( RH \) is average annual ambient relative humidity, percent; and \( f_{ps} \) is specified concrete compressive strength at transfer of prestress, psi (MPa).

The simplified equation was found to be a conservative estimate of prestress losses (Tadros et al. 2003) compared to full-scale test results and the results of the refined method. The simplified method, however, should not be used with noncomposite members, members with uncommon shapes or atypical levels of prestress, or unusual construction staging.

CHAPTER 6—LONG-TERM LOSSES: DETAILED METHODS

6.1—Scope

There are a variety of methods to estimate prestress losses when more detailed calculations are desired. For designs using assumed material properties, it is not always necessary to use detailed prestress loss calculations. Detailed methods can be categorized as time-step methods or age-adjusted effective modulus methods. It is also possible to account for additional effects on long-term losses due to differential shrinkage of concretes in composite systems, and thermal effects when casting composite decks or toppings.

Detailed methods are normally used for unusual design cases, such as very long span members or segmental bridges, to obtain a more accurate estimate of long-term losses using measured material properties. Detailed methods may also be a preferred option when unusual construction sequencing is used, such as spliced girders, which are initially pretensioned and then assembled and post-tensioned at the erection site. They can also be used to estimate prestress loss at any time during the life of the structure, as opposed to the simplified method, which can only provide an estimated loss at the end of service life.

6.2—Creep and shrinkage models

The ability of a detailed method to accurately estimate prestress loss is dependent on accurate models of creep and shrinkage of which several are widely used. ACI 209.2R presents four models for prediction of creep and shrinkage: 1) ACI 209R; 2) Bažant and Baweja (1995); 3) CEB-FIP model code (Comité Euro-International du Béton 1999); and 4) Gardner and Lockman (2001). Other models, such as Sakata (1993), Wendler et al. (2013), and AASHTO (2012), provide alternative calculation methods. There is no consensus on which model is the best predictor of time-dependent behavior and all have significant variability. ACI 209.2R provides comparisons of the predictions of the four presented models with a database of measurements, but does not endorse any one model.
The detailed models for estimation of prestress loss require the calculation of shrinkage strains and creep coefficients for the prestressed member concrete and, if the member has a composite deck or topping, the deck concrete. This guide does not review the models, but assumes the licensed design professional selects an appropriate model for the application.

### 6.3—Age-adjusted effective modulus approaches

#### 6.3.1 Age-adjusted effective modulus of elasticity

One approach to modeling the effects of time-dependent strains caused by creep is to adjust the concrete modulus of elasticity to account for both the elastic and time-dependent creep strains. Figure 6.3.1 illustrates this concept. The initial strain $\varepsilon(t_0)$ and the initial stress $\sigma(t_0)$ can be related by the initial elastic modulus $E_c$. When stress is sustained over time, additional strain will develop as a function of the creep coefficient $\phi(t, t_0)$ and the stress. Creep is a function of the time under consideration ($t$) and the time when the load was first applied ($t_0$). Thus, the final strain can be expressed as a function of the creep coefficient $\phi(t, t_0)$ (Ghali et al. 2012)

$$
\varepsilon(t) = \varepsilon(t_0) + \phi(t, t_0) \varepsilon(t_0) = \varepsilon(t_0)(1 + \phi(t, t_0))
$$

$$
= \frac{\sigma(t_0)}{E_c} \left( 1 + \phi(t, t_0) \right) = \frac{\sigma(t_0)}{E_c} \left[ 1 + \phi(t, t_0) \right] (6.3.1a)
$$

Another way to express this is to define the effective modulus of elasticity $E'_c$.

$$
E'_c = \frac{E_c}{1 + \phi(t, t_0)} (6.3.1b)
$$

This modulus is appropriate for calculating strains due to loads that are instantaneously applied, and then left in place for some time period.

Some stresses, such as those from prestress loss or the self-equilibrating stresses from differential shrinkage, develop slowly over time. One approach to this problem is to divide the stress into several increments and sum the elastic and creep strains for each increment of stress.

$$
\varepsilon(t) = \sum_{i=1}^{n} \frac{\Delta \sigma_i}{E_c(t_i)} \left[ 1 + \phi(t, t_i) \right] (6.3.1c)
$$

where the modulus of elasticity and creep coefficient are appropriate for the concrete age at the time of the initial increment of stress. The aging coefficient is typically taken as a number between 0.7 and 0.9. ACI 209R provides guidance in determining the appropriate aging coefficient as a function of the age when the stress begins to develop and the time over which the stress develops.

The age-adjusted effective modulus of elasticity, $E''_c$, is defined (Ghali et al. 2012)

$$
E''_c = \frac{E_c}{1 + \chi \phi(t, t_0)} (6.3.1d)
$$

where the modulus of elasticity and the creep coefficient are age-appropriate for the concrete age at the time of the initial increment of stress. The aging coefficient originally developed by Trost (1967). In this case, the total change in stress is applied in one step and the creep coefficient is modified to reflect that, as the stress slowly develops, the concrete is maturing and later increments of stress have less associated creep than earlier increments of stress. In this way, the total strain is expressed

$$
\varepsilon(t) = \frac{\sigma(t_0)}{E_c} \left[ 1 + \chi \phi(t, t_0) \right] (6.3.1e)
$$

This modulus is appropriate for calculating strains due to stresses that develop slowly over time. Two methods for estimating prestress loss outlined in this section use the age-adjusted effective modulus of elasticity in their formulations.  

#### 6.3.2 AASHTO LRFD refined method

The refined method for estimating prestress losses in the "AASHTO LRFD Bridge Design Specifications" (AASHTO 2012) was developed as part of National Cooperative Highway Research Program (NCHRP) Project 18-07 (Tadros et al. 2003). The method splits the estimation of long-term prestress losses into two time periods: 1) from the time of initial transfer of prestress to the time the composite deck is placed (subscript $i$); and 2) from deck placement to end of service (subscript $f$). For systems without a cast-in-place deck or topping, only one step, from initial to final (subscript $i$), is required. Although the method was developed for bridge girders, it is a general age-adjusted effective modulus method and can be applied to any pretensioned member.

The equation for time-dependent prestress loss is

$$
\Delta \sigma_{PT} = (\Delta \sigma_{S0} + \Delta \sigma_{CR1} + \Delta \sigma_{RD1}) + (\Delta \sigma_{S0} + \Delta \sigma_{CR2} + \Delta \sigma_{RD2} + \Delta \sigma_{sdeg}) (6.3.2a)
$$

where $\Delta \sigma_{PT}$ is long-term prestress loss, psi (MPa); $\Delta \sigma_{S0}$ is prestress loss due to shrinkage of beam concrete between
transfer and deck placement, psi (MPa); $\Delta \sigma_{\text{creep}}$ is prestress loss due to creep of beam concrete between transfer and deck placement, psi (MPa); $\Delta \sigma_{\text{rel}}$ is prestress loss due to relaxation of prestressing strands between time of transfer and deck placement, psi (MPa); $\Delta \sigma_{\text{shrink}}$ is prestress loss due to shrinkage of beam concrete between time of deck placement and final time, psi (MPa); $\Delta \sigma_{\text{creep-2}}$ is prestress loss due to relaxation of prestressing strands in composite section between time of deck placement and final time, psi (MPa); $\Delta \sigma_{\text{shrink-2}}$ is prestress loss due to shrinkage of deck in composite section, taken as a negative value in Eq. (6.3.2a), psi (MPa).

In Section 5.9.5.4 of AASHTO (2012), an equation is presented for each component of loss. NCHRP Report 496 (Tadros et al. 2003) presents the derivation of the equations. Each equation begins with the same assumption that there is perfect bond between the concrete and the steel; therefore, over time, the concrete at the strand level undergoes the same change in strain as the strand.

$$\Delta \varepsilon_p = \Delta \varepsilon_c$$  \hspace{1cm} (6.3.2b)

where $\Delta \varepsilon_p$ is change in strain in the prestressing steel; and $\Delta \varepsilon_c$ is change in strain in the concrete at the level of the prestressing steel.

The change in strain in the concrete is the time-dependent change, such as creep or shrinkage strain, minus the change in strain associated with prestress loss, $\Delta \sigma_p$. Prestress loss results in elastic and creep strain. Because prestress loss develops slowly over time, the change in concrete strain is taken as a negative value in Eq. (6.3.2a), psi (MPa).

For example, for the loss due to shrinkage the total change in strain at the strand level is calculated (Tadros et al. 2003)

$$\Delta \varepsilon_c = \varepsilon_{\text{shrink}}(t_d) - \left( \frac{\Delta \sigma_p}{A_g} + \frac{\Delta \sigma_p \varepsilon_p^2}{E_g} \right) \left( 1 + \chi \phi(t_f, t_d) \right) E_{cs}$$  \hspace{1cm} (6.3.2d)

where $\varepsilon_{\text{shrink}}(t_d)$ is shrinkage strain of the girder at the time the deck is placed; $t_d$ is time since end of cure to time of deck placement, days; $t_f$ is final time under consideration, days; and $t_d$ is time of initial prestress transfer, days.

Setting the change in concrete strain equal to the change in steel strain results in the equation

$$\frac{\Delta \sigma_p}{A_g E_p} = \varepsilon_{\text{shrink}}(t_d) - \left( \frac{\Delta \sigma_p}{A_g} + \frac{\Delta \sigma_p \varepsilon_p^2}{E_g} \right) \left( 1 + \chi \phi(t_f, t_d) \right) E_{cs}$$  \hspace{1cm} (6.3.2e)

To determine the prestress loss, all terms with $\Delta \sigma_p$ are collected to the left side of the equation and the change in force is divided by the strand area to arrive at a change in stress. The aging coefficient is set to 0.7 and the final form of the equation is

$$\Delta \sigma_{\text{shrink}} = \frac{\varepsilon_{\text{shrink}}(t_d) E_g}{1 + \frac{A_g}{E_g} \left( 1 + \frac{\varepsilon_p^2}{E_g} \right) \left( 1 + 0.7 \phi(t_f, t_d) \right)}$$  \hspace{1cm} (6.3.2f)

For simplification, a new term called a section modification factor is defined

$$K_{\text{shrink}} = \frac{1}{1 + \frac{A_g}{E_g} \left( 1 + \frac{\varepsilon_p^2}{E_g} \right) \left( 1 + 0.7 \phi(t_f, t_d) \right)}$$  \hspace{1cm} (6.3.2g)

The equation for change in prestress due to shrinkage that occurs from end of cure to time of deck placement can then be written as

$$\Delta \sigma_{\text{shrink}} = \varepsilon_{\text{shrink}}(t_d) E_p K_{\text{shrink}}$$  \hspace{1cm} (6.3.2h)

This approach is used for each type of prestress loss over each time interval.

The refined method includes the effect of deck shrinkage on changes in strand stress. When the deck concrete shrinks, after composite behavior has been established, the top of the girder restrains the shrinkage. In this way, the deck exerts a compressive force on the top of the girder, and the girder exerts an equal tensile force on the deck. This results in the girder deflecting downward and tension developing in the bottom of the girder. This also results in an increase in strand tension at the bottom of the beam. This is illustrated in Figs. 6.3.2a and 6.3.2b.

In the AASHTO (2012) approach, changes in prestress from deck and girder shrinkage are evaluated separately. For determining the increase in prestress due to deck shrinkage, the force exerted by the deck on the composite girder is determined

$$P_{\text{shrink}} = \varepsilon_{\text{shrink}}(t_d) A_g E_{cd}$$  \hspace{1cm} (6.3.2i)

where $P_{\text{shrink}}$ is force to fully restrain deck shrinkage, lb (N); $\varepsilon_{\text{shrink}}(t_d)$ is shrinkage of the deck concrete at final time under consideration; $A_g$ is area of composite concrete deck, in.$^2$ (mm$^2$); and $E_{cd}$ is modulus of elasticity of the composite deck, psi (MPa).

This force is assumed to be applied at the centroid of the deck on the composite girder. Knowing the force and location of application, the stress at the center of gravity of the prestress force can be calculated. Because this is a slowly developing force, the modulus of elasticity is divided by (1 + $\chi \phi(t_f, t_d)$) to reflect the relaxation of the force due to creep. The stress at the center of gravity of the prestress force is calculated

$$\frac{\Delta \sigma_p}{A_g E_p} = \left( \frac{\Delta \sigma_p}{A_g} + \frac{\Delta \sigma_p \varepsilon_p^2}{E_g} \right) \left( 1 + \frac{\varepsilon_p^2}{E_g} \right) \left( 1 + \chi \phi(t_f, t_d) \right)$$  \hspace{1cm} (6.3.2j)
Deck Shrinkage

Resulting stresses and deformation due to girder restraining deck shrinkage

Fig 6.3.2a—Effect of differential shrinkage on girder deflection.

Fig. 6.3.2b—Self-equilibrating stresses due to differential shrinkage.

\[
\Delta f_{def} = \frac{e_{sh}(t_f)A_cE_c}{\left(1+\chi(t_f,t_f)\right)} \left(\frac{1}{A_{comp}} - \frac{e_p e_d}{I_c}\right) \quad (6.3.2j)
\]

where \(\Delta f_{def}\) is concrete stress at the center of gravity of the prestress force due to the differential shrinkage force, psi (MPa); \(A_{comp}\) is transformed area of the composite section, in.\(^2\) (mm\(^2\)); \(I_c\) is transformed moment of inertia of the composite section, in.\(^4\) (mm\(^4\)); \(e_p\) is eccentricity of the prestress relative to the composite section centroid, in. (mm); \(e_d\) is eccentricity of the centroid of the deck relative to the composite section centroid, in. (mm); and \(\chi(t_f,td)\) is creep coefficient of the deck concrete at final time due to loads placed immediately following the end of moist cure.

Then, using the same approach as with other losses, the change in steel strain and the change in concrete strain at the level of the prestress are equated, and the gain in prestress is determined. The full equation is

\[
\Delta f_{pss} = \frac{E_p}{E_c} \Delta f_{def} \left(1+0.7\chi(t_f,td)\right) \quad (6.3.2k)
\]

where \(\phi(t_f,td)\) is creep coefficient of the beam concrete at the final time under consideration, for loads applied at the time of deck placement.

If a designer chooses to calculate this increase in prestress force, the increase in tension at the bottom of the girder should also be considered when checking allowable stresses. Equation (6.3.2j) can be altered to calculate the concrete tension at the bottom of the beam. The term \(e_{pc}\) is replaced by \(y_{con}\), the distance from the centroid of the composite section to the bottom of the section, to arrive at the tensile stress at the bottom of the beam due to deck shrinkage.

Typically, the tensile stresses at the bottom of the beam due to differential shrinkage are ignored. The stresses tend to be relatively small depending on several parameters, such as girder age at the time the deck is placed, relative cross-sectional areas of the deck and girder, shrinkage properties of the deck, and amount of deck reinforcing steel. Therefore, ignoring these stresses does not typically lead to any obvious distress in the beam.

However, if a designer chooses to take advantage of the increase in prestress force due to differential shrinkage, the detrimental effects, which may be greater than the advantages, should also be considered.

6.3.3 A general age-adjusted effective modulus approach—A second refined method, which can be used to evaluate prestress loss, is similar to the AASHTO (2012) refined method. Both methods use an age-adjusted effective modulus approach to account for the effect of the slowly changing force in the prestress. In the AASHTO approach, the effects of creep, shrinkage, and relaxation are investigated independently. In this approach (Menn 1990), all time-dependent sources of loss are considered in a single formulation. Figure 6.3.3a shows the basic model to predict prestress losses before deck placement.

There are six unknowns and six equations that can be written to solve for the unknowns. The equations are shown.

Equations of Internal Equilibrium:

\[
\Delta N_b + \Delta N_{pe} = 0 \quad (6.3.3a)
\]

\[
\Delta M_b + \Delta N_{pe} \cdot e_p = 0 \quad (6.3.3b)
\]

Constitutive Equations:

\[
\Delta e_p = \frac{(\Delta N_{pe} - \Delta N_{rel})}{A_p E_p} \quad (6.3.3c)
\]

\[
\Delta e_b = \frac{N_{pe}}{A_p E_p} \Delta N_{rel} + \frac{\Delta N_b}{A_p E_p} \left(1 + \chi(t_f,td) + e_{sh}(t)\right) \quad (6.3.3d)
\]

Compatibility Equations:

\[
\Delta \kappa = M_{pe} \phi(t_f,td) \frac{1}{I_p E_p} + \frac{M_p}{I_p E_p} \left(1 + \chi(t_f,td)\right) \quad (6.3.3e)
\]

\[
\Delta \kappa = \Delta e_p - \Delta \kappa \cdot e_p \quad (6.3.3f)
\]

where \(\Delta N_b\) is change in force in the beam, lb (N); \(\Delta N_{pe}\) is change in prestress force, lb (N); \(\Delta M_b\) is change in moment in the beam, in.-lb (N-mm); \(\Delta e_p\) is change in prestress strain; \(\Delta e_b\) is change in strain in the beam at the gross section centroid; \(\Delta \kappa\) is change in curvature, rad./in. (rad./mm);
$M_o^c$ is initial creep-producing moment in the girder, in.-lb (N-mm); $N_o^c$ is initial creep-producing force in the girder, lb (N); and $\Delta N_{relax}$ is change in prestress force due to relaxation (no associated strain), lb (N).

All unknowns are assumed to be positive as the equations are written. A positive notation means tension or lengthening, and a negative notation means compression or shortening. A positive curvature or moment causes compression at the top of the beam. When all unknowns are determined, a positive value will reflect the correct assumption. The cross-sectional properties of the beam are based on the net section; however, the gross properties are an acceptable approximation. The creep coefficient and shrinkage strain can be calculated with any acceptable model and reflect the time interval from prestress release to deck placement.

For a member with a composite deck, the same approach is used for the time interval from deck placement to final time. Figure 6.3.3b presents the unknowns for this interval. There are now 11 unknowns, so 11 equations are written.

**Equations of Internal Equilibrium:**

$$\Delta N_b + \Delta N_d + \Delta N_{ps} + \Delta N_{sd} = 0 \quad (6.3.3g)$$

$$\Delta M_g + \Delta M_b - \Delta N_d \cdot a_b + \Delta N_{ps} \cdot e_p - \Delta N_{sd} \cdot a_b = 0 \quad (6.3.3h)$$

**Constitutive Equations:**

$$\Delta e_d = \frac{\Delta N_d}{E_d A_d} \left(1 + \chi \phi_d(t_f, t_d)\right) + e_{sd}(t_f)$$

$$\Delta v = \frac{\Delta N_e}{E_e A_e} \left(1 + \chi \phi_e(t_f, t_d)\right) + (e_{sd}(t_f) - e_{sd}(t_f))$$

$$\Delta e_b = \frac{N_o^c}{E_o^c A_o^c} \left(\phi(t_f, t_d) - \phi(t_f, t_d)\right)$$

$$\Delta e_p = \frac{\Delta N_{ps}}{A_{ps} E_{ps}}$$

$$\Delta e_{sd} = \frac{\Delta N_{sd}}{A_{sd} E_{sd}}$$

$$\Delta \kappa = \frac{M_o^c}{E_o^c I_o} \left(\phi(t_f, t_d) - \phi(t_f, t_d)\right)$$

$$+ \frac{M_{deck}}{E_d I_d} \left(\phi(t_f, t_d) + \Delta M_{sd} \left(1 + \chi \phi_d(t_f, t_d)\right)\right)$$

$$\Delta \kappa = \frac{\Delta M_{sd}}{E_d I_d} \left(1 + \chi \phi_d(t_f, t_d)\right)$$

**Compatibility Equations:**

$$\Delta N_{ps} = \Delta N_{sd} - \Delta \kappa \cdot e_p$$

$$\Delta e_d = \Delta e_{sd}$$

where $e_p$ is eccentricity from the centroid of the beam (gross section) to the centroid of the prestress, in. (mm); $a_b$ is eccentricity from the centroid of the beam (gross section) to the centroid of the deck (also the centroid of the deck reinforcing steel), in. (mm); $\Delta N_{relax}$ is change in force in the deck reinforcement, lb (N); $\Delta N_{d}$ is change in force in the deck concrete, lb (N); $\Delta M_{d}$ is change in moment in the deck concrete, in.-lb (N-mm); $E_o^c$ is modulus of elasticity of the deck concrete, psi (MPa); $A_o^c$ is area of the composite concrete deck, in.² (mm²); $\phi_d(t_f, t_d)$ is creep coefficient of the deck concrete at final time due to loads at time of deck placement; $\chi$ is aging coefficient; $e_{sd}(t_f)$ is shrinkage strain in the deck at final time; $\phi(t_f, t_d)$ is creep coefficient of the beam at final time for loads placed at time of deck casting; $\phi(t_f, t_d)$ is creep coefficient of the beam at final time for loads placed at time of prestress transfer; $\phi(t_f, t_d)$ is creep coefficient of the beam at final time for loads placed at time of prestress transfer; $\phi(t_f, t_d)$ is instant prestress loss considering the effect of deck reinforcement and deck shrinkage. $E_o^c$ is initial creep-producing moment in the girder, in.-lb (N-mm); $N_o^c$ is initial creep-producing moment in the girder, lb (N); $M_o^c$ is moment in the beam just before deck placement, in.-lb (N-mm); $M_{deck}$ is moment in the beam due to deck weight, in.-lb (N-mm); $e_{sd}(t_f)$ is total shrinkage strain of the beam at the time of analysis; $e_{sd}(t_d)$ is shrinkage strain in the girder at the time the deck is placed; $\Delta \kappa$ is change in strain in the deck steel; $I_d$ is moment of inertia of the deck, in.⁴ (mm⁴); $E_o^c$ is modulus of elasticity of the deck steel, psi (MPa); and $A_o^c$ is area of deck steel, in.² (mm²).

When these equations are solved simultaneously, the time-dependent prestress loss considering the effect of deck reinforcement and deck shrinkage can be determined. Note that the given system of equations is based on the centroid of the deck reinforcement at the centroid of the deck. If this is not the case, the equations can be easily altered to reflect the actual location of the reinforcement. Instantaneous changes, such as elastic shortening losses and increases in strain at the time of deck placement, are calculated using the transformed cross-sectional properties of the girder. This general method, which is adaptable to all types of composite construction, can evaluate the prestress losses in precast, post-tensioned deck panels after they have been made composite with either a steel or concrete beam. Other reinforcement layers, such as
6.4—Incremental time-step method

Incremental time-step approaches to prestress loss calculations are based on superposition of elastic and creep strains resulting from increments of stress. Figure 6.4 presents the concept graphically. A varying load history is represented as a series of load increments that are left in place over time. Each load increment has associated elastic and creep strains. At any point in time, the total strain can be calculated as the summation of the elastic and creep strains for all loads plus shrinkage strains.

Guidance for time-step analyses for estimating prestress losses is found in textbooks and other sources (Nilson 1987; Naaman 2012; PCI Committee on Prestress Losses 1975; Gilbert and Ranzi 2011). The life of the structure is a summation of time increments that includes important loading stages expected to occur over the structure’s life. Typically, early-age increments are relatively short and later increments are longer because larger changes occur earlier in the life of the structure. For each time increment, the following five-step calculations assume all tendons are grouped together at their centroid. The subscript for the beginning of the time step is “n-1” and the end of the time step is “n.”

Step 1: Calculate the increment of creep and shrinkage for the time step—Use the selected model to determine the creep coefficient and the shrinkage strain at the beginning and end of the time step: \( \phi(t_{n-1}, f_0), \phi(t_f, f_0), \varepsilon_{shb}(t_{n-1}), \varepsilon_{shb}(t_f) \) then calculate the increment of creep and shrinkage for the time step as the differences between the starting and ending values.

Step 2: Calculate the initial elastic strains on the cross section—Based on the axial forces and moments on the cross section at the beginning of the time step, the cross-sectional properties, and the modulus of elasticity, calculate the load-induced strain distribution through the depth of the section. The concrete strain at the center of gravity of the prestressing force is of primary importance; however, the method can also be used to determine changes in curvature and deflection.

Step 3: Calculate time-dependent strain at the center of gravity of the prestressing force—Multiply the elastic strain at the center of gravity of the prestressing force by the increment of the creep coefficient \( \phi(t_{n-1}, f_0) - \phi(t_f, f_0) \) to determine the increment of creep strain, \( \Delta \varepsilon_c \). Add to that the increment of shrinkage strain, \( \Delta \varepsilon_{shb} = \varepsilon_{shb}(t_f) - \varepsilon_{shb}(t_{n-1}) \). Multiply the sum of the time-dependent strains by the modulus of elasticity of the prestressing steel and add to this the increment of relaxation loss for the time step. This is the gross increment of prestress loss for the time step.

Step 4: Calculate elastic rebound due to prestress loss—Calculate the change in stress at the center of gravity of the prestressing force due to the prestress loss, and divide by the concrete modulus of elasticity to determine the associated change in strain. Multiply this strain by the steel modulus to determine the change in prestress. This will be a tensile stress, which is also called prestress recovery or elastic rebound. Subtract this stress from the loss computed in Step 3 to arrive at the net increment of prestress loss for the time step.

Step 5: Determine creep-producing forces for the start of the next time step—Adjust the prestress force and moment to reflect the prestress loss over the previous time step. This new force and moment are used to calculate the elastic strains at the beginning of the next time step.

6.4.1 Variations in approach—A variety of approaches to the time-step method have been proposed (Waldron 2004;
ACI 435R; Moustafa 1986; Gutierrez et al. 1996; Tadros et al. 1975). The simplest, especially for beams that are not made composite, is to use only one creep model throughout the calculations. This means the ultimate creep coefficient is based on the assumption that all loads are applied at one point in time. The initial elastic strains are based on the modulus of elasticity at the time the first loads are applied.

Another approach is to treat each increment of prestress loss as a newly applied load. The creep coefficient for this load is calculated based on the maturity of the concrete at the end of the time step. Then, the elastic strains at the beginning of the next time step are the sum of the initial elastic strains based on the modulus of elasticity at release and the strains associated with the prestress loss based on the concrete modulus of elasticity at the start of the time step. This approach is theoretically more correct, but also more time-consuming.

6.5—Computer Programs

Several commercially available software programs include calculation of prestress losses or general time-dependent effects in concrete. The designer should determine which method of loss calculation and creep and shrinkage functions are being used in the program to know the strengths and weaknesses of the method for the individual design application.

6.6—Effects of Deck Temperature during Casting of Composite Deck or Topping

When fresh concrete is placed and begins to cure, the concrete heats up due to the hydration reaction between the water and cement. When deck concrete is placed on top of a beam, this temperature increase causes the top flange of the beam to expand relative to the bottom flange. The thermal gradient induces self-equilibrating stresses through the depth of the beam. Hypothetically, if the concrete did not harden once it achieved its peak temperature, and the beam was allowed to return to its original length, the stresses would be eliminated. However, as the deck concrete hardens, it can begin to take on load that locks in stresses produced by the deck heating.

The effect of deck curing on prestress losses in composite beams can be examined qualitatively; however, there are unknown or uncertain values. They include early-age concrete properties such as modulus and coefficient of thermal expansion, thermal gradient in the top of the beam, and development of composite action through bonding of the deck concrete to the top of the beam. As with the evaluation of thermal effects during beam casting, a quantitative evaluation is difficult.

The first assumption to make is with the thermal gradient. As reported by Roberts-Wollmann et al. (1995), based on studies of thermal gradients in match cast segments of segmental bridges, the temperature gradient dissipates exponentially over the first 12 in. (305 mm) of the heated element. A linear approximation is made to simulate this heat dissipation for ease of calculation. Depending on casting conditions, a reasonable assumption is that the concrete temperature will rise to approximately 110 to 120°F (43 to 49°C), and the beam will be at approximately ambient temperature. The self-equilibrating stresses from this gradient are calculated using the common approach of calculating moment and axial forces required to fully restrain thermal movements.

\[ P = \int E_c \cdot \alpha_c \cdot b(y) \cdot T(y) \, dy \]  \hspace{1cm} (6.6a)

\[ M = \int E_c \cdot \alpha_c \cdot b(y) \cdot T(y) \, y \, dy \]  \hspace{1cm} (6.6b)

where \( \alpha_c \) is coefficient of thermal expansion of the concrete; \( y \) is distance from the centroid of the cross section, in. (mm); \( T(y) \) is temperature at depth \( y \), °F (°C); and \( b(y) \) is width of cross section at depth \( y \), in. (mm).

The stresses at any location \( y \) through the depth of the beam are calculated as

\[ \sigma(y) = T(y) \cdot \alpha_c \cdot E_c \cdot P \cdot M_y \cdot I \]  \hspace{1cm} (6.6c)

To calculate the effect of temperature gradient on the beam, the deck concrete modulus of elasticity is calculated over a 2-day period. Measurements have shown a 2-day period is the time required for the concrete to reach ambient temperature, according to an unpublished thesis by Marston (2010). For a simple analysis, it is assumed that when the deck concrete reaches its maximum temperature, the deck has begun to harden and the modulus of elasticity increases from that point until it reaches its 28-day modulus. If measured values of the modulus are available, a curve can be fit to represent the modulus at any time after maximum temperature. An equation that fits the data (ACI 209R) is

\[ E(t) = \frac{E_{28}}{(a + \beta t)} \]  \hspace{1cm} (6.6d)

where: \( E(t) \) is modulus of elasticity at any time \( t \), psi (MPa); \( a \) is constant; and \( t_k \) is age of concrete, in days; and \( \beta \) is constant such that

\[ \beta = \frac{(28 - a)}{28} \]  \hspace{1cm} (6.6e)

If data are not available, ACI 209R provides typical values for the constants so a curve can be approximated.

Once the equation for the modulus is determined, the modulus is calculated at several intervals over the 2-day cooling period every 6 hours for the first and second day. This determines the curvature of the girder, the stress induced in the girder, and the camber of the girder over this time period.

After the modulus of elasticity has been determined for the appropriate times, the temperature gradient is determined for the same times. A constant temperature for the deck and linear gradient through the first 12 in. (305 mm) of the beam is a simple and reasonable assumption.
After the deck concrete has reached its maximum temperature, the deck begins to harden and the temperature cools. A composite section is created and the reduction in temperature causes the girder camber to reduce. The deck begins resisting load because it began to harden at the maximum girder camber, inhibiting the girder from returning to its original camber immediately after the deck was placed. This causes permanent stresses to be locked in the deck and girder. As the deck gets stiffer over time, more load is resisted by the deck, which further inhibits camber reduction.

As the deck cools, the stress calculation becomes dependent on the change in temperature from the maximum. At this point, the composite section properties of the girder with the deck should be calculated using the modulus of elasticity of each at the time of analysis, resulting in section properties that vary at each time interval. Because the deck is composite, it is included in the moment and axial force equations.

By calculating the moment and axial force on the girder during deck cooling, the locked-in stresses at the bottom of the composite section are calculated. This analysis predicts the tensile stress induced at the bottom of the girder, which reduces the cracking moment when the girder is loaded. The change in the prestress force can also be determined.

The analysis shows that there is additional tension at the bottom of the beam and a slight increase in prestress tension. This effect is accounted for by AASHTO (2012) and ACI 318-11 by using a somewhat smaller cracking stress $6\sqrt{f'_c}$ (for $f'_c$ in psi) $(0.5\sqrt{f'_c}$ [for $f'_c$ in MPa]) for the $V_c$, term in shear calculations for pretressed beams as opposed to $7.5\sqrt{f'_c}$ (for $f'_c$ in psi) $(0.63\sqrt{f'_c}$ [for $f'_c$ in MPa]), which is used to calculate the cracking moment for non-pretressed beams.

The effect of deck curing temperature on camber changes and prestress changes can be investigated using the methods as outlined in this section; however, the effect is small and the effort unnecessary for typical prestressed beams.

CHAPTER 7—VARIABILITY OF LOSS CALCULATIONS

7.1—Objective

Chapter 7 explores the range of calculated prestress losses based on the material properties and the modulus of elasticity values in ACI 318. The PCI Design Handbook (PCI 2010) is used as a baseline for calculated prestress losses and assessing the difference between calculated and measured losses. Chapter 7 is intended to be a general study on variation in losses, which enables the designer to appreciate the range of variation associated with the loss calculations.

7.2—Scope

This chapter addresses three variations of prestress losses:

1. The variation of the basic material properties, specifically modulus of elasticity and creep, from theoretical calculations is addressed.

2. Parametric studies of selected losses on a double-tee beam and a hollow core slab are completed to evaluate the possible range of variation from standard calculations.

3. A case study of shrinkage variations is presented. These studies provide an indication of the range of measured prestress losses as they may differ from calculated losses. These variations are common to all prestressed members. Variation from anchor seating and friction are more associated with post-tensioned members and relaxation, while common to all members, has a smaller contribution to total losses.

7.3—Contributions to prestress loss

Prestress losses are inherently variable and are derived from numerous factors as discussed in 7.2. Figure 7.3a illustrates this variability and is based on the assessment of losses in a pretensioned double-tee beam, a post-tensioned slab, and a post-tensioned beam presented in Chapter 8. In particular, note the variation among the various effects. For example, shrinkage varies from 15 percent of the losses in the double-tee beam to 53 percent of the losses in the post-tensioned beam, although the difference in the total magnitude of the shrinkage losses between the members is only 800 psi (5.50 MPa). Thus, the magnitude of the shrinkage losses is only slightly greater in the post-tensioned beam, while the percentage of total loss due to shrinkage in the post-tensioned beams is much greater. The total losses of post-tensioned slab and beam are much less than that of the pretensioned double-tee beam, primarily due to elastic shortening and creep losses. The loss due to elastic shortening is the largest of all losses in the pretensioned beam, but is practically negligible for both the post-tensioned slab and beam. The stress at the centroid of the prestressing force—$f_{or}$ for the bonded examples and $f_{or}$ for the unbonded slab example—is much larger in the double-tee ($1250 \text{ psi} [8.62 \text{ MPa}]$) than in the post-tensioned slab ($160 \text{ psi} [1.10 \text{ MPa}]$) or beam ($300 \text{ psi} [2.07 \text{ MPa}]$). The higher concrete compressive stress leads to much higher losses due to creep.

Based on these examples, it can be inferred that variability in shrinkage properties will have a larger influence as a percentage on the post-tensioned members than the pretensioned beam. However, variability in the modulus of elasticity and creep properties will have a larger influence on a pretensioned beam as compared to a post-tensioned member.

Studies have been conducted on the variation of losses and probabilistic studies on the variation are available (Steinberg 1995; Gilbertson and Ahlborn 2004). Of particular interest is the Gilbertson and Ahlborn study, as it compares alternative computation approaches and the potential variation in results. Figure 7.3b shows the variation in prestress loss for an I-girder. The figure illustrates two important issues: 1) The results of calculation of losses vary with the method selected; and 2) Even within one methodology, there is a variation of results. The details of each method are discussed in the Gilbertson and Ahlborn paper.

Gilbertson and Ahlborn (2004) examine the variation in all sources of prestress loss. In this chapter, two sources of prestress loss are considered: elastic shortening and creep. Shrinkage variations are examined later in a short case study. Losses due to anchor seating are omitted because they are often compensated by overstressing in pretensioning opera-
Fig. 7.3a—Example of the contribution to losses for different concrete elements.

Fig. 7.3b—Variation in prestress losses for an I-girder (Gilbertson and Ahlborn 2004). (Note: 1 ksi = 6.89 MPa.)

7.4—Modulus of elasticity

Modulus of elasticity directly impacts the initial elastic shortening of a member and indirectly affects the creep contribution to loss. The following discussion explores variation in modulus in detail and the corresponding impact of variation on the loss calculations.

7.4.1 Factors affecting modulus of elasticity—Concrete modulus of elasticity, \( E_c \), is one of the predominant factors affecting prestress loss calculation. Modulus of elasticity varies with concrete constituent materials and varies with time. Accordingly, significant variations in estimated losses are rooted in the variation of modulus of elasticity. The accurate estimate of prestress losses is dependent on an accurate valuation of the modulus of elasticity. Factors that influence the modulus of elasticity are the type and volume fraction of coarse and fine aggregate, moisture state of the specimen, loading conditions, and stiffness and porosity of the cement paste. Knowledge of these factors has long been in the literature (Hirsch 1962). Because all the factors are usually related to concrete compressive strength and unit weight, empirical equations to calculate \( E_c \) have been developed in terms of \( f_{pc}^\prime \) and unit weight. Different modulus of elasticity equations are considered because the value of \( E_c \) varies with the equation used.

7.4.2 Equations for modulus of elasticity—A multitude of equations for concrete modulus of elasticity are available in the literature. Comparisons of six different equations of \( E_c \) are examined with respect to compressive strength (Mehta and Monteiro 1993). The values for \( E_c \) based on concrete strength are summarized in Tables 7.4.2a and 7.4.2b, and Fig. 7.4.2a.

Each equation gives different modulus of elasticity values for a given compressive strength. Selecting the ACI 318-08 Eq. (1) for modulus places the user in the midrange of these predictive equations.

Each empirical equation was developed by correlating a large database of measured concrete strength with modulus test data. To apply any of the equations, measured concrete strength at a given age should be used for \( f_{pc}^\prime \). However, for either design or investigation, the licensed design professional generally has no information on in-place concrete strength. Thus, common practice uses the specified concrete compressive strength of concrete. This practice can lead to considerable discrepancy in the corresponding value for the modulus of elasticity \( E_c \), because the concrete actually produced in the field or in a prestressing plant will typically exceed the specified strength by a substantial margin to meet the acceptance criteria for a given project. Thus, while the specified properties may be well established, properties at the time of load application vary.

Storm (2011) studied the properties of plant-produced concrete. They conducted tests on multiple sets of specimens for each of 382 prestressed bridge girders of different types produced by nine prestressing plants in six mid-Atlantic and southeastern states. They found that, at prestress transfer, the measured concrete compressive strength was approximately 25 percent higher than the specified transfer strength (Fig. 7.4.2b). Similarly, the measured concrete compressive strength was approximately 45 percent higher than the specified strength at 28 days. Accordingly, Storm (2011) recommends that the specified concrete strength \( f_{pc}^\prime \) should be multiplied by 1.25 to compute the modulus of elasticity at prestress transfer, and \( f_{pc}^\prime \) be multiplied by 1.45 to compute the modulus of elasticity at 28 days.

7.4.3 Aggregate influence—The modulus of elasticity equations do not account for factors other than unit weight and compressive strength. Other factors that influence the value of modulus of elasticity include proportion of coarse aggregate in the concrete and aggregate properties. Mehta and Monteiro (1993) as well as Mindess et al. (2003) show that the modulus of elasticity value is affected by the coarse aggregate content and type. The modulus of elasticity equations are compared using quartzite aggregate concrete as the baseline. For other types of aggregate, the modulus of elasticity is obtained by multiplying \( E_c \) with a correction factor as shown in Table 7.4.3.
Table 7.4.2a—Equations for modulus of elasticity*

<table>
<thead>
<tr>
<th>Source</th>
<th>Equation</th>
<th>( E_s ) and ( f'_c ) in psi and ( w_c ) in lb/ft³</th>
<th>( E_s ) and ( f'_c ) in MPa and ( w_c ) in kg/m³</th>
<th>Equation no.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI 318-08</td>
<td>( E_s = 33 w_c^{0.5} f'_c )</td>
<td>( E_s = 0.043 w_c^{0.5} f'_c )</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td>Empirical Eq. (1) (Mehta and Monteiro 1993)</td>
<td>( E_s = 6,000,000(1 + (2000/f'_c)) )</td>
<td>( E_s = 41,370(1 + (13.8/f'_c)) )</td>
<td>(2)</td>
<td></td>
</tr>
<tr>
<td>Empirical Eq. (2) (Mehta and Monteiro 1993)</td>
<td>( E_s = 1,800,000 + 460/f'_c )</td>
<td></td>
<td>(3)</td>
<td></td>
</tr>
<tr>
<td>ACI 363 R-92</td>
<td>( E_s = \frac{40,000}{f'_c + 1} + (10745 w_c^{0.23}) )</td>
<td>( E_s = 3320 f'_c^{0.6} + 6895 w_c^{0.32} )</td>
<td>(4)</td>
<td></td>
</tr>
<tr>
<td>Ahmad and Shah (1982)</td>
<td>( E_s = 3.37 w_c^{0.5} f'_c^{0.3} )</td>
<td>( E_s = 21,500 f'_c^{0.6} (10 w_c^{0.23}) )</td>
<td>(5)</td>
<td></td>
</tr>
<tr>
<td>CEB-FIP (2010)</td>
<td>( E_s = 3.120,000 f'_c^{0.6} (10 w_c^{0.23}) )</td>
<td></td>
<td>(6)</td>
<td></td>
</tr>
</tbody>
</table>

*Note: \( f'_c \) is the compressive strength and \( w_c \) the unit weight of concrete.

Table 7.4.2b—Comparison of different modulus of elasticity equations*

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
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<td>3000</td>
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<td>3,600,000</td>
<td>3,180,000</td>
<td>3,357,000</td>
<td>3,718,000</td>
<td>4,000,000</td>
</tr>
<tr>
<td>4000</td>
<td>3,605,000</td>
<td>4,000,000</td>
<td>3,640,000</td>
<td>3,714,000</td>
<td>4,082,000</td>
<td>4,300,000</td>
</tr>
<tr>
<td>5000</td>
<td>4,031,000</td>
<td>4,286,000</td>
<td>4,100,000</td>
<td>4,028,000</td>
<td>4,389,000</td>
<td>4,600,000</td>
</tr>
<tr>
<td>6000</td>
<td>4,415,000</td>
<td>4,500,000</td>
<td>4,560,000</td>
<td>4,312,000</td>
<td>4,657,000</td>
<td>4,900,000</td>
</tr>
<tr>
<td>7000</td>
<td>4,769,000</td>
<td>4,667,000</td>
<td>5,020,000</td>
<td>4,573,000</td>
<td>4,896,000</td>
<td>5,200,000</td>
</tr>
<tr>
<td>8000</td>
<td>5,098,000</td>
<td>4,800,000</td>
<td>5,480,000</td>
<td>4,817,000</td>
<td>5,114,000</td>
<td>5,500,000</td>
</tr>
<tr>
<td>9000</td>
<td>5,407,000</td>
<td>4,909,000</td>
<td>5,940,000</td>
<td>5,045,000</td>
<td>5,313,000</td>
<td>5,800,000</td>
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<td>10,000</td>
<td>5,700,000</td>
<td>5,000,000</td>
<td>6,400,000</td>
<td>5,261,000</td>
<td>5,498,000</td>
<td>6,100,000</td>
</tr>
</tbody>
</table>

*Note: All units are psi, concrete unit weight is 145 lb/ft³, 1000 psi = 6.896 MPa.

Fig. 7.4.2a—Comparison of different modulus of elasticity equations.

In Fig. 7.4.3, elastic moduli are plotted versus \( f'_c \) after taking aggregate type factor into account by applying the correction factor to the \( E_s \) as computed by the ACI 318 equation. Note that the value of \( E_s \) is highest for the concrete with basalt as a coarse aggregate, which is followed by quartz and limestone, and at the least with sandstone.

7.4.4 Volume of coarse aggregate—Modulus of elasticity values are also affected by the volume fraction of the coarse aggregate. Figure 7.4.4 presents the variation of modulus of elasticity with respect to variation in the volume content of various aggregates. Examining the aggregates typically used in concrete, the modulus tends to increase with increasing coarse aggregate volume. This effect is not reflected in any of the modulus equations.

7.4.5 Weight of aggregate—Unit weight effects generally follow the strength corrections for aggregate types and

![Graph showing the variation of modulus of elasticity with respect to aggregate types.](image-url)
Fig. 7.4.3—Comparison of modulus of elasticity for different aggregate types (adapted from Myers and Carrasquillo (1999)). (Note: 1000 psi = 6.896 MPa.)

therefore are not cumulative effects. The term of $33w^{0.5}$ used in Eq. (1) (Table 7.4.2a) provides the correction for unit weight.

7.4.6 Conclusions for modulus of elasticity—The following conclusions are based on selecting the equation in ACI 318-11 as the “correct” baseline. Empirical Eq. (1) (Table 7.4.2a) overestimates the modulus of elasticity value for $f_{c}'$ range up to 6000 psi (41.4 MPa), while Eq. (2) (Table 7.4.2b) overestimates the value for $f_{c}'$ above 6000 psi (41.4 MPa). Ahmad and Shah (1982) and the CEB-FIP (2010) equations in this guide give higher $E_c$ values for the entire range of $f_{c}'$.

A correction factor to account for aggregate type in the modulus of elasticity equation is close to the difference due to the weight of coarse aggregate; therefore, mixture weight plays a prominent role in the determination of $E_c$. However, just accounting for the aggregate weight instead of the volume fraction or stiffness of coarse aggregate can overestimate the modulus calculation. From the discussion in 7.3 and 7.4, the licensed design professional can reasonably expect to see variations of up to 30 percent between calculated and measured modulus.

7.5—Creep

7.5.1 Factors affecting creep—Concrete creep is a time-dependent phenomenon of continued deformation under sustained load. It depends on several factors, including the volume of cement paste, water-cementitious material ratio ($w/cm$), type and volume of aggregates, concrete age at the time of loading, prestress level, age at which the concrete is prestressed, and member geometry. A full discussion of creep is found in ACI 209.1R and ACI 209.2R. Among these factors, creep is most affected by paste volume. Many of the interactions are similar to those affecting the modulus of elasticity (Glucklich and Ishai 1962). For example, higher aggregate volume, harder aggregates, and higher strength tend to reduce creep. Rather than reiterate all of the interactions within the concrete mixture that affect creep, licensed design professionals typically reduce a complex problem to a single creep coefficient.

7.5.2 Creep coefficient—For the purpose of computing losses, the effect of creep is often computed using a creep coefficient $C_c$, which is the ratio of the long-term creep deformation to the initial elastic deformation. Thus, creep is directly tied to the modulus of elasticity. Typical creep coefficients range between 1 and 3.5 with a value of 2.35 recommended for many normal weight concrete mixtures.

7.6—Variational analysis

To appreciate how losses can vary from calculated values, a series of loss calculations are made varying only the modulus of elasticity and the creep coefficient. The loss calculations follow the recommendations given earlier in this report. Two prestressed members are examined: a 32 in. (813 mm) deep, 10 ft (3.05 m) wide double-tee beam with a 2 in. (51 mm) topping and an 8 in. (203 mm) deep hollow core slab. Section properties are taken from the PCI Design Handbook (PCI 2010). Calculations consider that the number of strands varies with span length.

The study first examines a 60 ft (18.3 m) long double-tee beam with the only variable being the modulus of elasticity. Figure 7.6a shows the range of losses possible by using alternate equations for modulus. The horizontal axis is the modulus of elasticity rather than the concrete strength, and the equations used to calculate the modulus are indicated.

The assessment of modulus of elasticity indicated that a variation of up to 30 percent was possible using existing data. That range of variation translates to a ±10 percent variation in losses compared to the losses calculated using Eq. (1) from ACI 318-08.

The effect of creep on the loss calculation is more pronounced. Figures 7.6b and 7.6c indicate the loss calculations for variation of creep coefficient for both double-tee
beam and hollow core members with $f'_c$ varying from 5000 to 10,000 psi (34.5 to 69.0 MPa). Using a creep coefficient of 2.0 as a baseline and an increment of ±0.5 for the creep coefficient, the variation in losses for double-tee beams range from ±23 percent with the low modulus to ±17 percent for the high modulus. The loss variations for hollow core slabs are similar (Fig. 7.6c).

While the examples are based on Eq. (1) (ACI 318-08) from Table 7.4.2a, it is instructive to consider the effect of a change in the source data for the calculations. If the CEB-FIP (2010) modulus equation (Eq. (6) in Table 7.4.2a) is used for the modulus of elasticity and a normal creep coefficient is used to estimate the creep strains, then the initial modulus of elasticity would have been overestimated when compared to a prediction using the ACI equation. The initial elastic shortening would then be lower. When multiplied by the creep coefficient, the result would predict a lower total deformation and correspondingly lower losses.

### 7.7—Shrinkage case study

The variational analysis examines only modulus of elasticity and creep. Figure 7.3a indicated that shrinkage has a large impact on losses. Shrinkage variation is not as easy to quantify as a variation in creep coefficient, so a case study is used to illustrate variations due to shrinkage. Lark et al. (2004) observed the short- and long-term behavior of two bridge girders on the Cogan and Grangetown Viaducts in the United Kingdom. Both were cast during summer, but Grangetown Viaduct was cast when the humidity level was 23 percent higher than when Cogan was cast. The difference in the strain behavior of these two bridges is shown in Fig. 7.7.

Figure 7.7 indicates substantial differences between predicted and measured strains in the structure, even when sophisticated modeling tools are used. A nearly 3-to-1 difference in strains is observed between the Cogan and Grangetown Viaducts at an age of 80 days, as noted in the figure.

Bridge girders have a smaller surface-to-volume ratio than flat slabs. Thus, it is reasonable to expect that variation of shrinkage strains in flat slab construction will be at least as large as beams, as is reflected in Fig. 7.3a.

### 7.8—Self-consolidating concrete

Self-consolidating concrete (SCC) is coming into wider use and its properties, as they affect losses, are changing with newer mixture designs. Because most SCC mixtures sacrifice coarse aggregate content in preference for fine materials, including both fine aggregate and cementitious materials, the modulus of elasticity of SCC is expected to be less than that of normal concrete of similar strength. Accordingly, the licensed design professional may expect that SCC will be affected by creep strains to a much larger extent than

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**Fig. 7.6a**—Prestress losses for double tee with variation in modulus of elasticity. (Note: 1000 psi = 6.896 MPa.)

**Fig. 7.6b**—Double-tee beam losses for various creep coefficients. (Note: 1000 psi = 6.896 MPa.)

**Fig. 7.6c**—Hollow core slab losses for various creep coefficients. (Note: 1000 psi = 6.896 MPa.)
normal concrete. A full discussion of the effects of SCC, their components, and the impact on variation of losses is beyond the scope of this section. An awareness of the impact of SCC on modulus, creep, and shrinkage is important to understand loss calculations. Additional information on losses in SCC is found in Levy et al. (2010), Schindler et al. (2007), Long and Khayat (2011), Gross et al. (2007), and Kavanaugh et al. (2008).

7.9—Conclusions

This chapter examines the range of a small number of factors affecting loss computations. The computations suggest that variations in losses between a baseline calculation and performance do exist. Figure 7.9, from Gilbertson and Ahlborn (2004), shows variations also exist for different calculation methods for a box girder. Variation in losses resulting from variations in modulus of elasticity and creep alone can approach ±45 percent if each factor is considered separately and then combined. From Fig. 7.3b and 7.9, the variation in losses from the mean ranges from 15 to 30 percent for the PCI general and simplified approaches discussed earlier. Further examples of loss calculations are shown in Chapter 8.

Fig. 7.7—Comparison of theoretical and measured strains in the Grangetown and Cogan Viaducts (adapted from Lark et al. (2004)).

Fig. 7.9—Estimated losses for box girder from all sources (Gilbertson and Ahlborn 2004). (Note: 1 ksi = 6.895 MPa.)

Note that the maximum variation of all factors occurring simultaneously is unlikely. While variations for each factor might seem large, the variation in total losses ranges from 8 to 15 percent of the typical calculated losses. The variation between theoretical and measured loss decreases with improved knowledge of the concrete properties.
CHAPTER 8—EXAMPLES

This chapter presents several examples of prestress loss calculations. The first example is a pretensioned double-tee beam, and several methods are used to calculate losses. This example illustrates the differences in the various methods in terms of result and the computational effort required to perform the calculations. Two post-tensioned examples are then presented, an unbonded slab and a beam with bonded tendons. Only the simplified approach is used for these examples. Finally, an example is presented illustrating the effects of temperature changes prior to prestress transfer in a pretensioned beam.

8.1—Pretensioned double-tee beam

Example 8.1 presents several approaches to the calculation of prestress losses in a typical double-tee beam. The elastic shortening losses are calculated with four methods and the long-term losses are calculated with four methods. At the end of the section, results are compared.

### 8.1.1 Givens and problem statement

The beam in this problem is a double-tee beam with the designation 10LDT 32 + 2; dimensions as shown in Fig. 8.1.1.

![Fig. 8.1.1—Double-tee beam.](image)

Span \( \ell = 70 \text{ ft} \) (21.3 m)

No superimposed dead load except topping

\( RH = 75 \text{ percent} \)

Section properties (untopped):

- \( A_g = 615 \text{ in.}^2 \) \((= 3.97 \times 10^5 \text{ mm}^2)\)
- \( I_g = 59,720 \text{ in.}^4 \) \((= 2.49 \times 10^{10} \text{ mm}^4)\)
- \( y_{bot} = 21.98 \text{ in.} \) \((= 558.3 \text{ mm})\)
- \( S_b = 2,717 \text{ in.}^3 \) \((= 4.45 \times 10^7 \text{ mm}^3)\)
- \( V/S = 1.69 \text{ in.} \) \((= 42.9 \text{ mm})\)

Precast concrete—sand-lightweight with unit weight:

- \( w_c = 115 \text{ lb/ft}^3 \) \((= 1842 \text{ kg/m}^3)\)

Self-weight of the double-tee beam:

- \( w_{self} = 491 \text{ lb/ft} \) \((= 7170 \text{ N/m})\)

Weight of the normalweight concrete topping placed after erection (assume age of double-tee is 90 days):

- \( w_t = 250 \text{ lb/ft} \) \((= 3650 \text{ N/m})\)

Properties of the sand-lightweight concrete:

- \( f_{c'} = 5000 \text{ psi} \) \((= 34.5 \text{ MPa})\)
- \( E_{c'} = 2,900,000 \text{ psi} \) \((= 20.0 \times 10^3 \text{ MPa})\)
- \( f_{c'} = 3500 \text{ psi} \) \((= 24.1 \text{ MPa})\)
- \( E_{c'i} = 2,400,000 \text{ psi} \) \((= 16.5 \times 10^3 \text{ MPa})\)

Properties of the prestressing steel:

- \( (12) \) 1/2-in. diameter, 270,000 psi, low-relaxation strands \((12.7 \text{ mm dia.}, 1860 \text{ MPa})\)
- \( A_{ps} = 12(0.153) = 1.836 \text{ in.}^2 \) \((= 12 (98.7) = 1184 \text{ mm}^2)\)
- \( E_{ps} = 28,500,000 \text{ psi} \) \((= 197 \times 10^3 \text{ MPa})\)
- \( f_{pu} = 0.75f_{pu} \)
Depressed at midspan with the following eccentricities:
*At the ends of the beam:*
\[ e_e = 12.81 \text{ in. (325.4 mm)} \]
*At midspan, single drape point:*
\[ e_c = 18.73 \text{ in. (475.7 mm)} \]

**Problem statement:** Determine total prestress loss.

8.1.2 **Elastic shortening losses**—This section presents four approaches to calculate elastic shortening.

8.1.2.1 **Gross section approximation**—For depressed strands with a single drape point, the critical section is approximately at 0.4\( \ell \). The moments, eccentricity, and prestress force at this location are first calculated.

\[ M = w(0.4\ell)(\ell - 0.4\ell)/2 = 0.120w\ell^2 \]

The moment due to the self-weight of the beam at the 0.4\( \ell \) point is

\[ M_g = 0.12(491)(70^2) = 289,000 \text{ ft-lb} = 3,468,000 \text{ in}-\text{lb} \]

The eccentricity \( e \) at 0.4\( \ell \) is

\[ e = 12.81 + 0.8(18.73 - 12.81) = 17.55 \text{ in. (445.8 mm)} \]

Assume compensation for anchorage seating loss during prestressing, so the force in the strand immediately before release is

\[ P_j = 0.75A_{psf} = 0.75(1.836)(270,000) = 371,800 \text{ lb (1,654,000 N)} \]

Determine the concrete stress at the tendon level, \( f_{c,	ext{ir}} \), making the assumption that the force after elastic shortening is 90 percent of the jacking force, as suggested in Zia et al. (1979).

\[ f_{c,	ext{ir}} = K_{c,	ext{ir}} \left( \frac{P_j}{A_g} + \frac{P_e e_r^2}{I_g} \right) \frac{M_g e_r}{I_g} \]

\[ f_{c,	ext{ir}} = 0.9 \left( \frac{371,800 \text{ lb}}{615 \text{ in.}^2} + \frac{371,800 \text{ lb} \times (17.55 \text{ in.})^2}{59,720 \text{ in.}^4} \right) - \frac{3,468,000 \text{ in.-lb} \times 17.55 \text{ in.}}{59,720 \text{ in.}^4} \]

\[ f_{c,	ext{ir}} = 1251 \text{ psi (8.625 MPa)} \]

\[ \Delta f_{\text{els}} = f_{c,	ext{ir}} \frac{E_p}{E_c} = 1251 \text{ ksi} \times \frac{28,500,000 \text{ psi}}{2,400,000,000 \text{ psi}} = 14,900 \text{ psi (103 MPa)} \]

8.1.2.2 **Gross section with iteration**—Begin with the elastic shortening loss calculated in (8.1.2.1), and determine what fraction of the original force the loss represents; calculate a new value of \( K_{c,	ext{ir}} \)

\[ K_{c,	ext{ir}} = \frac{\text{stress after transfer}}{\text{jacking stress}} = \frac{202,500 \text{ psi} - 14,900 \text{ psi}}{202,500 \text{ psi}} = 0.926 \]

Use the new value of \( K_{c,	ext{ir}} \) to calculate a new value of \( f_{c,	ext{ir}} \)

\[ f_{c,	ext{ir}} = 0.926 \left( \frac{371,800 \text{ lb}}{615 \text{ in.}^2} + \frac{371,800 \text{ lb} \times (17.55 \text{ in.})^2}{59,720 \text{ in.}^4} \right) - \frac{3,468,000 \text{ in.-lb} \times 17.55 \text{ in.}}{59,720 \text{ in.}^4} \]

\[ f_{c,	ext{ir}} = 1316 \text{ psi (9.07 MPa)} \]

Use the new \( f_{c,	ext{ir}} \) to calculate a new elastic shortening loss:

Check if this is consistent with \( K_{c,	ext{ir}} = 0.926 \)

\[ K_{c,	ext{ir}} = \frac{\text{stress after transfer}}{\text{jacking stress}} = \frac{202,500 \text{ psi} - 15,540 \text{ psi}}{202,500 \text{ psi}} = 0.923 \]
The new $K_{cr}$ is within 1 percent of the starting value, so the iteration can be concluded, or an additional iteration can be performed with the new $K_{cr}$.

$$f_{cr} = 0.923 \left( \frac{371,800 \text{ lb}}{615 \text{ in.}^2} + \frac{371,800 \text{ lb} \times (17.55 \text{ in.})^3}{59,720 \text{ in.}^4} \right) - \frac{3,468,000 \text{ in.-lb} \times 17.55 \text{ in.}}{59,720 \text{ in.}}$$

$$f_{cr} = 1309 \text{ psi (9.03 MPa)}$$

$$\Delta f_{pks} = f_{cr} \frac{E_p}{E_c} = 1309 \text{ psi} \frac{28,500,000 \text{ psi}}{2,400,000 \text{ psi}} = 15,540 \text{ psi (107.1 MPa)}$$

$$K_{cr} = \frac{\text{stress after transfer}}{\text{jacking stress}} = \frac{202,500 \text{ psi} - 15,540 \text{ psi}}{202,500 \text{ psi}} = 0.923$$

The new $K_{cr}$ matches the starting value, so the iteration is complete.

8.1.2.3 Closed form of iterative method—Elastic shortening losses are calculated using Eq. (4.3.3), which is

$$\Delta f_{pks} = e_M A_x - A_p f_{ph} \left( I_g + e_p^2 A_y \right)$$

$$\Delta f_{pks} = 17.55 \text{ in.} (3,468,000 \text{ in.-lb}) (615 \text{ in.}^2) - 1.836 \text{ in.}^2 (202,500 \text{ psi}) (59,720 \text{ in.}^4 + (17.55 \text{ in.})^2 (615 \text{ in.}^2))$$

$$\Delta f_{pks} = -15,550 \text{ psi (107.2 MPa)}$$

Note that the result is a negative number indicating a prestress loss.

8.1.2.4 Transformed section method—First the transformed area and moment of inertia are calculated:

$$A_t = A_x + A_p (n_p - 1)$$

$$n_p = \frac{E_p}{E_c} = \frac{28,500,000 \text{ psi}}{2,400,000 \text{ psi}} = 11.88$$

$$A_t = 615 \text{ in.}^2 + 1.836 \text{ in.}^2 (11.88 - 1) = 635.0 \text{ in.}^2 (4.10 \times 10^5 \text{ mm}^2)$$

$$c_{gt} = \frac{A_t y_g + A_p (n_p - 1) y_{ps}}{A_t}$$

$$c_{gt} = \frac{615 \text{ in.}^2 (21.98 \text{ in.}) + (1.836 \text{ in.}^2) (11.88 - 1)(4.43 \text{ in.})}{635.0 \text{ in.}^2} = 21.43 \text{ in. (544.3 mm)}$$

$$I_t = I_g + A_p (y_{gt} - c_{gt})^2 + A_p (n_p - 1)(y_{ps} - c_{gt})^2$$

$$I_t = 59,720 \text{ in.}^4 + 615 \text{ in.}^5 (21.98 \text{ in.} - 21.43 \text{ in.}) + (1.836 \text{ in.}^3) (11.88 - 1)(21.43 \text{ in.} - 4.43 \text{ in.})^2$$

$$I_t = 65,676 \text{ in.}^4 (2.73 \times 10^9 \text{ mm}^4)$$

Calculate the concrete stress at the strand level using the full jacking force, self-weight moment, and transformed section properties

$$f_{cr} = \frac{371,800 \text{ lb}}{635.0 \text{ in.}^2} + \frac{371,800 \text{ lb} \times (21.43 \text{ in.} - 4.43 \text{ in.)}^2}{65,676 \text{ in.}^4} - \frac{3,468,000 \text{ in.-lb} \times (21.43 \text{ in.} - 4.43 \text{ in.})}{65,676 \text{ in.}^4}$$

$$f_{cr} = 1324 \text{ psi (9.30 MPa)}$$
\[ \Delta \rho_{PES} = 1324 \text{ psi}(11.88) = 15,730 \text{ psi (108.5 MPa)} \]

Note that, using the iterative method in 8.1.2.2, with the net section properties of the beam, will give the same elastic shortening result as the transformed section method.

8.1.2.5 Summary of elastic shortening calculations for double-tee beam—Table 8.1.2.5 presents the results of the elastic shortening calculations using the four methods. Note that all losses are considered positive.

<table>
<thead>
<tr>
<th>Method</th>
<th>Section</th>
<th>Elastic shortening loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross section approximation</td>
<td>8.1.2.1</td>
<td>14,900 psi</td>
</tr>
<tr>
<td>Gross section approximation with iteration</td>
<td>8.1.2.2</td>
<td>15,500 psi</td>
</tr>
<tr>
<td>Closed form solution to iteration</td>
<td>8.1.2.3</td>
<td>15,550 psi</td>
</tr>
<tr>
<td>Transformed section method</td>
<td>8.1.2.4</td>
<td>15,700 psi</td>
</tr>
</tbody>
</table>

The gross section approximation method predicts less loss than the other three methods, which predict similar losses. This is due to the approximation of \( K_{cir} \) as 0.9, which is less than the actual value. Depending on the problem, this approximation may be larger or smaller than the more-refined methods. This is a source of error in the approximate method; however, the effort in the calculation is less than the other three. A designer can weigh the implications of the loss of accuracy compared to the lower level of effort required. Also recall, the accuracy of all of the methods is dependent on the modulus of elasticity, which is difficult to predict accurately.

8.1.3 Long-term losses—In this section, four approaches to the calculation of long-term losses are presented:
(a) Simplified method (Chapter 5)
(b) AASHTO (2012) refined method (6.3.2)
(c) General age-adjusted effective modulus method (6.3.3)
(d) Incremental time-step method (6.4)
The AASHTO (2012) simplified method (5.5) is not applicable because the topping is not composite.

8.1.3.1 Simplified method—The starting point for the simplified method is the simplest approach to the calculation of elastic shortening loss, the gross section approximation. So the values of \( f_{0b} \) and \( \rho_{PES} \) in the calculations are 1251 psi (8.63 MPa) and 14,900 psi (103.0 MPa), respectively.

First, the moment and stress at the tendon level due to the topping are calculated:
\[ M_{sd} = 0.12 \times 250 \text{ lb/ft}(70 \text{ ft})^2 = 147,000 \text{ ft-lb} = 1,764,000 \text{ in.-lb (199 \times 10^6 N-mm)} \]

\[ f_{ob} = \frac{M_{sd} e_x}{I_g} = \frac{1,764,000 \text{ in.-lb} \times 17.55 \text{ in.}}{59,720 \text{ in.}^4} = 518 \text{ psi (3.57 MPa)} \]

Calculate the prestress loss due to creep:
\[ \Delta \rho_{PES} = K_c (f_{ob} - f_{ob}) \frac{E_p}{E_c} = 1.6(1251 \text{ psi} - 518 \text{ psi}) \frac{28,500,000 \text{ psi}}{2,900,000 \text{ psi}} = 11,500 \text{ psi (79.3 MPa)} \]

Calculate the loss in prestress due to shrinkage:
\[ \Delta \rho_{PES} = 8.2 \times 10^{-6} K_a E_p \left(1 - 0.06 \frac{V}{S}\right)(100 - RH) \]
\[ \Delta \rho_{PES} = 8.2 \times 10^{-6} \times 1.0 \times 28,500,000 \text{ psi}(1 - 0.06 \times 1.69)(100 - 75) = 5300 \text{ psi (36.5 MPa)} \]

Calculate the prestress loss due to relaxation:
\[ \Delta \rho_{PES} = [K_{cr} - J(\Delta \rho_{PSH} + \Delta \rho_{PES} + \Delta \rho_{PES})] \]

From Table 5.4:
Using Eq. (5.4.2a) with:

\[ f_p / f_{pc} = 0.75 \]

\[ C = 1.0 \]

\[ \Delta f_{pc} = [5000 \text{ psi} - 0.04(5300 \text{ psi} + 11,500 \text{ psi} + 14,900 \text{ psi})] \times 1.0 = 3700 \text{ psi} (25.5 \text{ MPa}) \]

Sum the three losses to arrive at the long-term loss:

\[ \Delta f_{LT} = \Delta f_{CR} + \Delta f_{SH} = 11,500 \text{ psi} + 5300 \text{ psi} + 3700 \text{ psi} = 20,500 \text{ psi} (141.0 \text{ MPa}) \]

The total change in prestress from just before release to end of service, not including elastic gains from topping is:

\[ \Delta f_p = \Delta f_{ES} + \Delta f_{LT} = 14,900 \text{ psi} + 20,500 \text{ psi} = 35,400 \text{ psi} (244.0 \text{ MPa}) \]

The effective prestress at end of service is:

\[ f_{pe} = f_{pc,MC} - \Delta f_p = 202,500 \text{ psi} - 35,400 \text{ psi} = 167,100 \text{ psi} (1152 \text{ MPa}) \]

To calculate the stresses in the beam at the end of service, this tendon stress is used along with gross section properties. It is common practice to not include the increase in tendon stress associated with applied loads such as the topping or live loads in the calculation of \( f_{pc} \).

**8.1.3.2 AASHTO LRFD (2012) refined method** — The AASHTO LRFD refined method uses the elastic shortening losses and associated concrete compressive stresses as calculated with the transformed section method, so the values of \( f_{pc} \) and \( f_{pc,CS} \) in the calculations are 1324 psi (9.13 MPa) and 15,700 psi (108.2 MPa), respectively. AASHTO’s notation for \( f_{pc} \) is \( f_{pc,CR} \).

Assume topping is noncomposite.

Calculate creep coefficients (using the AASHTO LRFD (2012) Model)

\[ \phi(t,t) = 1.9k_kskhc(\phi^{0.118}) \]

For the creep due to initially applied loads considered at the time of topping placement:

\[ t_i = 1\text{-day steam cured} \]

\[ t = 90 \text{ days} \]

\[ k_s = 1.45 - 0.13(175) \geq 1.0 \]

\[ k_t = 1.45 - 0.13(1.69) = 1.23 \]

\[ k_{ks} = 1.56 - 0.008RH \]

\[ k_{ks} = 1.56 - 0.008(75) = 0.96 \]

\[ k_{f} = \frac{5}{1 + 3.5} = \frac{5}{1 + 3.5} = 1.11 \]

At 90 days:

\[ k_{ad} = \frac{t}{61 - 4f'_c + t} = \frac{90}{61 + 4(3.5) + 90} = 0.657 \]

\[ \phi(90,1) = (1.9)(1.23)(0.96)(1.11)(0.657)(1) = 1.636 \]

For the creep due to initially applied loads considered at the end of service:

All factors remain the same except the \( k_{ad} \) term is 1.0

\[ \phi(\infty,1) = (1.9)(1.23)(0.96)(1.11)(1.0)(1) = 2.49 \]

For the creep due to the topping, applied at 90 days, considered at the end of service:

There is no change in the \( k_s \) and \( k_t \) terms, and \( k_{ad} \) term is 1.0
Calculate shrinkage strains:

\[ \varepsilon_{sh} = k_k k_h k_d (0.48 \times 10^{-3}) \]

Shrinkage at 90 days:
The \( k, k_1, \) and \( k_{d} \) terms are the same as the first creep calculation

\[ k_h = 2.00 - 0.014RH = 2.00 - 0.014(75) = 0.95 \]

\[ \varepsilon_{ad}(90) = (1.23)(0.95)(1.11)(0.657)(0.48 \times 10^{-3}) = 0.409 \times 10^{-3} \]

Shrinkage at end of service:
All terms are the same as 90-day calculation, except \( k_{d} = 1.0 \)

\[ \varepsilon_{ad}(\infty) = (1.23)(0.95)(1.11)(1.0)(0.48 \times 10^{-3}) = 0.623 \times 10^{-3} \]

The AASHTO method separates the losses into two time periods: release to deck casting (\( id \)), and deck casting to end of service (\( df \)). The total time-dependent losses can be split into the following components:

\[ \Delta f_{id} = (\Delta f_{SR} + \Delta f_{CRCR} + \Delta f_{PRL})_{id} + (\Delta f_{SS} + \Delta f_{CD} + \Delta f_{PR} - \Delta f_{PS})_{df} \]

The following sections illustrate the calculation of each component.

**Losses due to shrinkage from release to deck placement, \( \Delta f_{SR} \)**

First, the section modification term, \( K_{id} \), is calculated:

\[ K_{id} = \frac{1}{1 + \frac{E_p A_p}{E_c A_c} \left( 1 + \frac{A_p g_p \phi}{A_c} \right) \left( 1 + 0.7 \phi_c (t_s, t_r) \right)} \]

\[ K_{id} = \frac{1}{1 + \frac{28,500,000}{2,400,000} \frac{1.836}{615} \left( 1 + \frac{615(17.55)}{59,720} \right) \left( 1 + 0.7 \cdot 2.49 \right)} = 0.72 \]

The AASHTO equation for losses due to shrinkage from release to placement is:

\[ \Delta f_{SR} = \varepsilon_{ad}(90) E_p K_{id} = 0.409 \times 10^{-3} \cdot (28,500,000 \text{ psi}) \cdot (0.72) = 8400 \text{ psi} \ (57.9 \text{ MPa}) \]

**Losses due to creep from release to deck placement, \( \Delta f_{CRCR} \)**

\[ \Delta f_{CRCR} = \frac{E_p}{E_c} f_{cr} \phi (t_s, t_r) K_{id} = \frac{28,500,000}{2,400,000} (1324)(1.636)(0.72) = 18,500 \text{ psi} \ (127.6 \text{ MPa}) \]

**Losses due to relaxation from release to deck placement, \( \Delta f_{PR} \)**

Calculate strand stress after elastic shortening losses:

\[ f_p = f_p \Delta f_{PR} - \Delta f_{PS} = 202,500 \text{ psi} - 15,700 \text{ psi} = 186,000 \text{ psi} \ (1288 \text{ MPa}) \]

\[ K_L = 30 \text{ for low-relaxation strands} \]

\[ \Delta f_{PR} = \frac{f_p}{K_L} \left( f_p - 0.55 \right) = \frac{186,800}{30} \left( \frac{186,800}{243,000} - 0.55 \right) = 1400 \text{ psi} \ (9.65 \text{ MPa}) \]
Losses due to shrinkage from deck placement to end of service, $L_{lfp SD}$
Because the topping is noncomposite, the section modification term is the same for both time intervals:

$$K_{sl} = K_{sf} = 0.72$$

Calculate the shrinkage from time of deck placement to end of service:

$$\epsilon_{sf} = \epsilon_{sf}(0) - \epsilon_{sf}(90) = (0.623 - 0.409) \times 10^{-3} = 0.214 \times 10^{-3}$$

$$\Delta f_{lfp SD} = \epsilon_{sf}E_fK_{sl} = (0.214 \times 10^{-3})(28,500,000)0.72 = 4400 \text{ psi (30.3 MPa)}$$

Losses due to creep from deck placement to end of service, $L_{lfp CD}$
There are two causes of creep during this time interval. First, there will be continued creep due to the initially applied loads: prestress and self-weight. In most cases, this creep will be compressive at the bottom of the beam, which will continue to reduce the strand stress. Second, there will be creep from the addition of the permanent load of the topping. This creep will be tensile, increasing the strand tension. The AASHTO (2012) method also includes applying the prestress loss in the first interval as a tensile force, and accounting for the tensile creep associated with the prestress loss. It can be argued that this is unnecessary, because using the age-adjusted modulus has already accounted for it. The method now combines a stepwise analysis with an age-adjusted approach, which is double-counting the creep associated with prestress loss. This example, however, presents the calculations as dictated by the AASHTO method.

Calculate the long-term loss from release to deck placement:

$$\Delta f_{lfp sl} = \Delta f_{lfp R} + \Delta f_{lfp CN} + \Delta f_{lfp RI} = 8400 \text{ psi} + 18,500 \text{ psi} + 1400 \text{ psi} = 28,300 \text{ psi (195.1 MPa)}$$

Calculate the moment at 0.4$t$ due to the topping (8.1.1)

$$M_{SL} = 147,000 \text{ ft-lb} = 1,764,000 \text{ in-lb (199,300 N-m)}$$

Apply the topping dead load moment and the prestress loss to the section and calculate the concrete stress at the strand level. Use the transformed section properties for the dead load, and the gross section properties for the prestress loss.

$$\Delta f_{cd} = \frac{(-28,300 \text{ psi})(1.836 \text{ in.}^2) + (-28,300 \text{ psi})(1.836 \text{ in.}^2)(17.55 \text{ in.})^2}{615 \text{ in.}^2} = \frac{-1,764,000 \text{ in.-lb}(17.02 \text{ in.})}{65,423 \text{ in.}^4} = -811 \text{ psi (-5.59 MPa)}$$

This is a tensile stress. Now calculate total change in prestress due to creep from deck placement to final time:

$$\Delta f_{PCD} = \frac{\Delta f_{CD}}{E_f} = \frac{E_f}{E_{gi}} \Delta f_{CD}$$

$$\Delta f_{PCD} = \frac{28,500,000 \text{ psi}}{4,200,000 \text{ psi}} (1324 \text{ psi})(2.49 - 1.636)(0.72) + \frac{28,500,000 \text{ psi}}{2,900,000 \text{ psi}} (-811 \text{ psi})(1.1)(0.72)$$

$$\Delta f_{PCD} = 3400 \text{ psi (23.4 MPa)}$$

Loss due to relaxation from deck placement to end of service, $L_{fR2}$
AASHTO allows the simplifying assumption that the relaxation loss from initial to deck placement is equal to the loss from deck placement to final.

$$\Delta f_{R2} = \Delta f_{R1} = 1400 \text{ psi (9.65 MPa)}$$

Gain in prestress force due to differential shrinkage of topping relative to girder
Because the topping is considered to be noncomposite, this calculation is not required. There is some interaction between the topping and the girder, which results in downward displacement and tension in the concrete at the strand level. Common practice is to ignore this interaction, because there is no reinforcement to develop composite action between the topping and
the beam. If the cohesion between the topping and the beam is broken, any stresses that may have built up due to differential shrinkage will be relieved. Also, if the topping is thin, the differential shrinkage stresses will be small.

**Elastic gain in prestress force due to topping**

To determine the effective prestress at end of service, all elastic gains are calculated. The concrete stress at the strand level due to topping is calculated:

\[ \Delta f_{\text{SDL}} = \frac{1,764,000 \text{ in.-lb} (17.02 \text{ in.})}{65,423 \text{ in.}^4} = -459 \text{ psi} (-3.16 \text{ MPa}) \]

Calculate the change in prestress corresponding to the prior change in concrete stress:

\[ \Delta f_{\text{SDL}} \frac{E_p}{E_c} \frac{f_{\text{SDL}}}{\Delta f_{\text{SDL}}} = \frac{28,500,000 \text{ psi}}{2,900,000 \text{ psi}} (-459 \text{ psi}) = -4500 \text{ psi} (-31.0 \text{ MPa}) \]

**Summary**

The total long-term change in prestress force is calculated as:

\[ \Delta f_{pLT} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}) + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2}) \]

\[ \Delta f_{pLT} = (8400 + 18,500 + 1400) + (4400 + 3400 + 1400 + 0) = 37,500 \text{ psi} (258 \text{ MPa}) \]

The total change in prestress from just before release to end of service, including elastic losses and gains is:

\[ \Delta f_p = \Delta f_{pES} + \Delta f_{pLT} + \Delta f_{pSDL} = 15,700 \text{ psi} + 37,500 \text{ psi} - 4500 \text{ psi} = 48,700 \text{ psi} (336 \text{ MPa}) \]

The effective prestress at end of service is:

\[ f_{\text{pE}} = f_{\text{pJACK}} - \Delta f_p = 202,500 \text{ psi} - 48,700 \text{ psi} = 153,800 \text{ psi} (1060 \text{ MPa}) \]

**8.1.4 Double-tee beam with general age-adjusted effective modulus method**

Using creep and shrinkage values from the previous example, six equations are written to solve for six unknowns. One of the unknowns is the change in the force in the prestressing strands.

**Equations of internal equilibrium**

\[ \Delta N_h + \Delta V_{ps} = 0 \]

\[ \Delta M_b + \Delta N_{ps} \cdot e_p = 0 \]

Using gross cross-sectional properties, \( e_p = 17.55 \text{ in.} (445.8 \text{ mm}) \), so:

\[ \Delta M_b + 17.55 \Delta N_{ps} = 0 \]

**Constitutive equations**

\[ \Delta e_p = \frac{\Delta N_{ps} - \Delta N_{\text{relax}}}{A_{ps} E_p} \]

The total relaxation is assumed to be 2800 psi (19.3 MPa) for 1.836 in.\(^2\) (1184 mm\(^2\)) of prestress, \(-5100 \text{ lb} (-22,700 \text{ N})\), so the equation becomes:

\[ \Delta e_p = \frac{(\Delta N_{ps} - (-5100 \text{ lb}))}{(1.836 \text{ in.}^2)(28,500,000 \text{ psi})} = \frac{\Delta N_{ps}}{52,326,000} + 97.5 \times 10^{-6} \]

\[ \Delta e_p = \frac{N_p(t,t_4)}{A_p E_p} + \frac{\Delta N_{ps}}{A_{ps} E_p} \left(1 + \chi(t,t_4)\right) + e_{\text{rel}}(t) \]
The initial force in the beam is the jacking force minus the elastic shortening loss, the cross-sectional properties are the gross properties (as a close approximation of net), and the shrinkage term is the total shrinkage from release to end of service:

\[
\Delta \varepsilon_n = \frac{-\left(202,500 \text{ psi} - 15,700 \text{ psi}\right)(1.836 \text{ in}^2)(2.49)}{(615 \text{ in}^2)(2,400,000 \text{ psi})} + \frac{\Delta N_n}{(615 \text{ in}^2)(2,400,000 \text{ psi})}
\]

\[
\Delta \varepsilon_n = \frac{M_{\text{b}}}{E_{\text{b}}} + \frac{M_{\text{s}}}{E_{\text{s}}} + \frac{\Delta M_{\text{i}}}{E_{\text{i}}}
\]

Note that three creep-producing moments are considered: 1) the initially applied prestress and self-weight moment; 2) the moment from superimposed dead load; and 3) the changing moment due to creep.

\[
M_{\text{s}} = \left(-2,599,000 \text{ in}-\text{lb}\right) + 3,436,000 = -2,599,000 \text{ in}-\text{lb} (294,000 \text{ N-m})
\]

\[
\Delta \varepsilon_n = \frac{-\left(2,599,000 \text{ in}-\text{lb}\right)2.49}{(59,720 \text{ in}^4)(2,400,000 \text{ psi})} + \frac{1,764,000 \text{ in}-\text{lb}(1.1)}{(59,720 \text{ in}^4)(2,400,000 \text{ psi})}
\]

Equation of strain compatibility

\[
\Delta \varepsilon_n = \Delta \varepsilon_p - \Delta \varepsilon_c
\]

These equations are put into a matrix form to solve for the six unknowns. The resulting values are:

\[
\Delta \varepsilon_n = -1069 \mu \varepsilon
\]

\[
\Delta \varepsilon_p = -11 \mu \varepsilon/\text{in.} (0.43 \mu \varepsilon/\text{mm})
\]

\[
\Delta \varepsilon_c = -1263 \mu \varepsilon
\]

\[
\Delta N_n = 71,200 \text{ lb} (317,000 \text{ N})
\]

\[
\Delta M_{\text{b}} = 1,250,000 \text{ in}-\text{lb} (141,000 \text{ N-m})
\]

\[
\Delta M_{\text{s}} = 71,200 \text{ lb} (-317,000 \text{ N})
\]

The total change in prestress is:

\[
\Delta f_p = 15,700 \text{ psi} + 38,800 \text{ psi} - 4500 \text{ psi} = 50,000 \text{ psi} (345 \text{ MPa})
\]

The effective prestress at end of service is:

\[
f_{\text{eff}} = f_p - \Delta f_p = 202,500 \text{ psi} - 50,000 \text{ psi} = 152,500 \text{ psi} (1051 \text{ MPa})
\]

8.1.5 Double-tee beam with stepwise method—Time steps were selected, with smaller initial intervals and longer intervals later in the life of the beam. The days chosen were 10, 30, 90, 100, 5000, and 20,000 days. The AASHTO creep and shrinkage model, which was used for the previous examples, was also used to compute creep coefficients and shrinkage strains at each time step. Creep coefficients were determined for the initially applied loads, and the overlay load applied at 90 days. The creep coefficients and shrinkage strains are presented in Table 8.1.5a.

<table>
<thead>
<tr>
<th>Days since casting</th>
<th>Total shrinkage strain, microstrain</th>
<th>Creep coefficient for loads applied at Day 1</th>
<th>Creep coefficient for loads applied at Day 90</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>109</td>
<td>0.437</td>
<td>NA</td>
</tr>
<tr>
<td>30</td>
<td>243</td>
<td>0.970</td>
<td>NA</td>
</tr>
<tr>
<td>90</td>
<td>409</td>
<td>1.636</td>
<td>NA</td>
</tr>
<tr>
<td>100</td>
<td>424</td>
<td>1.694</td>
<td>0.216</td>
</tr>
<tr>
<td>5000</td>
<td>617</td>
<td>2.467</td>
<td>1.091</td>
</tr>
<tr>
<td>20,000</td>
<td>622</td>
<td>2.490</td>
<td>1.098</td>
</tr>
</tbody>
</table>

The first step is to establish the initial conditions. Based on previous calculations using the iterative method with gross cross-sectional properties for elastic shortening losses, the initial prestress stress, after elastic shortening losses, is 186,900 psi (1289 MPa), and the initial compression at the 0.4f location at the prestress level is 1309 psi (9.03 MPa) compression.
The following steps are made for each time step, based on the initial conditions:

**Calculate creep strain for time step**

\[ \varepsilon_{cr} = \frac{f_{cp}}{E_{cs}} \theta = \frac{1309 \text{ psi}}{2,400,000 \text{ psi}} \times 0.437 = 238 \mu e \]

**Calculate reduction of strand tension due to creep and shrinkage**

\[ \Delta f_{PSL-CR-SH} = E_{ps}(\varepsilon_{cr} + \varepsilon_{sh}) = 28,500,000 \text{ psi}(0.000238 + 0.000109) = 9900 \text{ psi (68.3 MPa)} \]

**Calculate relaxation loss for the time step**

\[ \Delta f_{REL} = \frac{f_p}{1000} \left( \log \left( \frac{t}{24} \right) \right) \left( \frac{f_p - 0.55}{f_p} \right) = 186,900 \text{ psi} \left( \frac{\log (10 \times 24)}{45} \right) \left( \frac{186,900 \text{ psi}}{243,000 \text{ psi}} - 0.55 \right) = 2200 \text{ psi (15.2 MPa)} \]

**Calculate gross loss for this step**

\[ \Delta f_{PSL gross} = 9900 \text{ psi} + 2200 \text{ psi} = 12,100 \text{ psi (83.4 MPa)} \]

**Determine change in concrete stress at centroid of strand due to prestress loss**

\[ \Delta f_{LOSS} = \frac{12,100 \text{ psi} \times 1.836 \text{ in.}^2}{615 \text{ in.}^2} + \frac{12,100 \text{ psi} \times 1.836 \text{ in.}^2(17.55 \text{ in.})^2}{59,720 \text{ in.}^4} = 151 \text{ psi (1.04 MPa) (tension)} \]

**Calculate the associated increase in tendon tension**

\[ \Delta f_{preboued} = 0.151 \text{ ksi} \times \frac{28,500,000 \text{ psi}}{2,400,000 \text{ psi}} = 1800 \text{ psi (12.4 MPa)} \]

**Calculate net loss for this step**

\[ \Delta f_{net} = 12,000 \text{ psi} - 1800 \text{ psi} = 10,300 \text{ psi (71.0 MPa)} \]

This loss is subtracted from the initial prestress to arrive at the prestress at the end of the time step: 186,900 psi – 10,300 psi = 176,600 psi (1218 MPa). With this steel stress, the associated concrete stress is determined. These are the starting values for the next time step. The calculations are repeated using the increment of creep and shrinkage for each time step. After the topping is placed, the creep associated with the new load is also included in the loss calculations. The results of the calculations are presented in Table 8.1.5b.

### Table 8.1.5b—Time-step prestress losses

<table>
<thead>
<tr>
<th>Start of time step ( f_p, \text{ psi (MPa)} )*</th>
<th>Day 10</th>
<th>Day 30</th>
<th>Day 90</th>
<th>Day 100</th>
<th>Day 5000</th>
<th>Day 20,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>186,900 (1289)</td>
<td>176,600 (1218)</td>
<td>166,700 (1149)</td>
<td>155,400 (1071)</td>
<td>155,300 (1071)</td>
<td>146,300 (1009)</td>
<td></td>
</tr>
<tr>
<td>Start of time step ( f_{ps}, \text{ psi (MPa)} )*</td>
<td>-1309 (-9.03)</td>
<td>-1181 (-8.14)</td>
<td>-1057 (-7.29)</td>
<td>-916 (-6.32)</td>
<td>-915 (-6.31)</td>
<td>-803 (-5.54)</td>
</tr>
<tr>
<td>Creep strain, ( \varepsilon_{cr} )</td>
<td>-238</td>
<td>-262</td>
<td>-293</td>
<td>12</td>
<td>-157</td>
<td>-7</td>
</tr>
<tr>
<td>Shrinkage strain, ( \varepsilon_{sh} )</td>
<td>-109</td>
<td>-133</td>
<td>-167</td>
<td>-15</td>
<td>-193</td>
<td>-4</td>
</tr>
<tr>
<td>Loss due to creep (CR) + shrinkage (SH), psi (MPa)</td>
<td>-9900 (-68.3)</td>
<td>-11,300 (-77.9)</td>
<td>-13,100 (-90.3)</td>
<td>-100 (-0.69)</td>
<td>-10,000 (-68.9)</td>
<td>-300 (-2.07)</td>
</tr>
<tr>
<td>Relaxation loss, psi (MPa)</td>
<td>-2200 (-15.2)</td>
<td>-300 (-2.07)</td>
<td>-200 (-1.38)</td>
<td>-0 (-0.0)</td>
<td>-500 (-3.45)</td>
<td>-100 (-0.69)</td>
</tr>
<tr>
<td>Gross loss for time step, psi (MPa)</td>
<td>-12,100 (-83.4)</td>
<td>-11,600 (-80.0)</td>
<td>-13,300 (-91.7)</td>
<td>-100 (-0.69)</td>
<td>-10,500 (-72.4)</td>
<td>-400 (-2.76)</td>
</tr>
<tr>
<td>Elastic rebound, psi (MPa)</td>
<td>1800 (12.4)</td>
<td>1700 (11.7)</td>
<td>2000 (13.8)</td>
<td>0 (0)</td>
<td>1600 (11.0)</td>
<td>100 (0.7)</td>
</tr>
<tr>
<td>Net loss for time step, psi (MPa)</td>
<td>-10,300 (-71.0)</td>
<td>-9900 (-68.3)</td>
<td>-11,300 (-77.9)</td>
<td>-100 (-0.7)</td>
<td>-8900 (-61.4)</td>
<td>-300 (-2.1)</td>
</tr>
<tr>
<td>End of time step ( f_p, \text{ psi (MPa)} )</td>
<td>176,600 (1218)</td>
<td>166,700 (1149)</td>
<td>155,400 (1071)</td>
<td>155,300 (1071)</td>
<td>146,300 (1009)</td>
<td>146,000 (1007)</td>
</tr>
<tr>
<td>End of time step ( f_{ps}, \text{ psi (MPa)} )</td>
<td>-1181 (-8.14)</td>
<td>-1057 (-7.29)</td>
<td>-916 (-6.32)</td>
<td>-915 (-6.31)</td>
<td>-803 (-5.54)</td>
<td>-799 (-5.51)</td>
</tr>
<tr>
<td>Concrete stress at level of strand due to topping psi (MPa)</td>
<td>460 (3.17)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Does not include stress changes due to topping.
The total time-dependent prestress loss is 40,900 psi (282 MPa), with elastic shortening losses and gain due to topping, the effective prestress is:

\[ f_{pe} = 202,500 - 15,600 - 40,900 + 4,500 = 150,900 \text{ psi (1040 MPa)} \]

Table 8.1.5c presents a summary of the results of the various methods to estimate prestress losses, and compares to the simplified approach presented in 8.1.1.

**Table 8.1.5c—Summary of results of loss calculations for example**

<table>
<thead>
<tr>
<th>Method</th>
<th>Elastic shortening, psi (MPa)</th>
<th>Time-dependent, psi (MPa)</th>
<th>Gain from topping, psi (MPa)</th>
<th>Total change, psi (MPa)</th>
<th>Effective prestress, psi (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simplified method</td>
<td>-14,900 (-102.7)</td>
<td>-20,500 (-141.3)</td>
<td>—</td>
<td>-35,400 (-244.1)</td>
<td>167,100 (1152)</td>
</tr>
<tr>
<td>AASHTO LRFD (2012) method</td>
<td>-15,700 (-108.2)</td>
<td>-37,500 (-258.0)</td>
<td>4500 (31.0)</td>
<td>-48,700 (-335.8)</td>
<td>153,800 (1060)</td>
</tr>
<tr>
<td>Age-adjusted effective modulus</td>
<td>-15,700 (-108.2)</td>
<td>-38,800 (-267.5)</td>
<td>4500 (31.0)</td>
<td>-50,000 (-355.7)</td>
<td>152,500 (1051)</td>
</tr>
<tr>
<td>Stepwise</td>
<td>-15,600 (-107.6)</td>
<td>-40,900 (-282.0)</td>
<td>4500 (31.0)</td>
<td>-51,600 (-355.8)</td>
<td>150,900 (1040)</td>
</tr>
</tbody>
</table>

Although the range of difference is on the order of 10 percent, the simplified method predicts considerably smaller loss than the other three methods. This is primarily because the creep loss calculation uses the 28-day modulus rather than the modulus at release, and the elastic shortening loss is smaller because no iteration was performed.

### 8.2—Post-tensioned slab with unbonded tendons

This example presents calculation of prestress losses in a typical one-way post-tensioned flat slab with unbonded monostrand tendons. The initial losses are calculated based on the methods in Chapter 4 and long-term losses are calculated using the simplified method described in Chapter 5.

**8.2.1 Given and problem statement**—This problem considers a six-span, one-way slab spanning 18 ft (5.49 m) between 14 in. (356 mm) x 34 in. (864 mm) cast-in-place concrete beams, as shown in Fig. 8.2.1

![Fig. 8.2.1—Slab dimensions and prestressing.](image)

Slab thickness = 5 in. (127 mm)

\[ RH = 80 \text{ percent} \]

Concrete properties: Normalweight

\[ f_{c'} = 3,000 \text{ psi (20.68 MPa) (at transfer)} \]

\[ f_{c'}' = 4,000 \text{ psi (27.58 MPa)} \]

\[ w_c = 150 \text{ lb/ft}^3 (2403 \text{ kg/m}^3) \]

\[ E_c = 3,122,000 \text{ psi (21,530 MPa)} \]

\[ E_c = 3,605,000 \text{ psi (24,860 MPa)} \]

Concrete age at stressing = 3 days
Strand properties:
Low-relaxation, unbonded system
Strand diameter = 1/2 in. (12.7 mm)

Required uniform tendon final effective force in slab = 9000 lb/ft (131,300 N/m) or one tendon spaced at 3 ft on center (0.91 m)
assuming a final effective force of 27,000 lb (120,100 N) per tendon.
Strand area = 0.153 in.² (98.7 mm²)

\[ E_p = 28,500,000 \text{ psi} (196,500 \text{ MPa}) \]
\[ f_{pu} = 270,000 \text{ psi} (1862 \text{ MPa}) \]

Jacking stress = 0.8 \times 270,000 \text{ psi} (1862 \text{ MPa}) = 216,000 \text{ psi} (1489 \text{ MPa})

Angular friction coefficient = 0.07 \text{ (rad⁻¹)}
Wobble friction coefficient = 0.001 \text{ (ft⁻¹)} (0.00328 \text{ m⁻¹})
Anchor set = 0.25 in (6.4 mm)
Reverse parabola at L/12
Assume stressing from one end (left end)

Problem statement: Determine initial friction, anchor set and elastic shortening losses and long-term creep, shrinkage, and relaxation losses.

8.2.2 Friction losses and elongation—Friction and anchor set calculations are calculated using the approach described in Section 4.4 where the “entire tendon is idealized as several individual parabolic segments.” The eccentricity at each parabolic vertex is calculated with the following expression (Eq. (8.2.2a)) (variables used shown in Fig. 8.2.2a)

\[ e_i = \frac{e_i - 1(x_i + 1 - x_i) + e_i + 1(x_i - x_i - 1)}{(x_i + 1 - x_i - 1)} \]  
(8.2.2a)

![Fig. 8.2.2a—Location of inflection points.](image)

The angular deviation is then computed for each segment (Eq. (8.2.2b)) and (Fig. 8.2.2b)

\[ \theta_j = \tan^{-1} \left( \frac{e_{j+1} - e_j}{x_{j+1} - x_j} \right) \]
(8.2.2b)

![Fig. 8.2.2b—Tangent at a point on a parabolic segment.](image)
The exponential in Eq. (8.2.2c) is computed. Multiplied by the jacking stress, it gives the tendon stress at the end of each segment before anchor set:

\[ \sigma_{\text{before set}} = \sigma_{\text{jacking}} e^{-(\beta x)} \]  

(8.2.2c)

Elongation of each segment before anchor set is computed with the average stress along the segment

\[ \text{elongation}_i = 12 \times \frac{1}{2} \left( \sigma_i + \sigma_{i-1} \right) x (x_i - x_{i-1}) + \text{elongation}_{i-1} \]  

(8.2.2d)

Table 8.2.2 provides the friction loss parameters calculated at various locations along the tendon length. The variables shown in the table are as defined in this section.

| Table 8.2.2—Friction and anchor set losses, and elongations |

<table>
<thead>
<tr>
<th>Vertex</th>
<th>( e_i ) eccentricity</th>
<th>( e_i - e_{i-1} ) cm</th>
<th>( x_i ) m distance from stressing end</th>
<th>( x_i - x_{i-1} ) m</th>
<th>( \theta_i ) rad segment angular deviation</th>
<th>( a_i ) rad angular deviation</th>
<th>( \exp(-k \times (l_{x_i} - a_x \times \rho)) ) stress in percent of stressing end</th>
<th>Stress before set, ksi</th>
<th>Stress after set, ksi</th>
<th>Elongation, in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stressing end</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.50</td>
<td>0.00</td>
<td>0.0</td>
<td>0.0</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>216.0</td>
<td>188.1</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>2.38</td>
<td>0.13</td>
<td>1.5</td>
<td>1.5</td>
<td>0.014</td>
<td>0.014</td>
<td>0.000</td>
<td>215.5</td>
<td>188.6</td>
<td>0.14</td>
</tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.75</td>
<td>0.63</td>
<td>9.0</td>
<td>7.5</td>
<td>0.028</td>
<td>0.028</td>
<td>0.028</td>
<td>213.6</td>
<td>190.4</td>
<td>0.81</td>
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<td>4</td>
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<td>7.5</td>
<td>0.049</td>
<td>0.069</td>
<td>0.079</td>
<td>211.4</td>
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<td>1.48</td>
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<td>0.111</td>
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<td>0.500</td>
<td>0.500</td>
<td>197.9</td>
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<td>0.555</td>
<td>0.555</td>
<td>196.8</td>
<td>196.8</td>
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<td>0.611</td>
<td>0.611</td>
<td>195.8</td>
<td>195.8</td>
<td>4.83</td>
</tr>
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<td>Midspan 4</td>
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<td>1.00</td>
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<td>63.0</td>
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<td>0.666</td>
<td>0.666</td>
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<td>193.6</td>
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<td>0.722</td>
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</tr>
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</table>

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2.3 Anchor set losses—The force lost in the tendon due to anchor set is represented in Fig. 8.2.3 by the area between the stress curves before and after anchor set. Knowing that the stress slopes are opposite, the anchor set length and the maximum stress $f_{max}$ are determined numerically. First, the segment with the maximum stress is identified, then the position is determined inside that segment. In this example, the anchor set length is 38.4 ft (11.7 m) and the maximum stress $f_{max}$ in the tendon is 202,200 psi (1394 MPa).

For tendons with many curvature reversals and low friction, such as slabs, a hand calculation is cumbersome. The simplified method shown in 4.4.2 can be used. For this example, the simplified method will give an anchor set length of 41.5 ft (12.7 m) and a maximum stress of 201,900 psi (1392 MPa). The anchor set is longer because the assumption of linear friction neglects that most friction occurs on supports.

![Fig. 8.2.3—Tendon stress diagram along tendon length.](image)

8.2.4 Elastic shortening—The elastic shortening loss is calculated using the simplified method described in Section 4.5.

$$\Delta f_{ES} = K_c f_{pe} f_{pul} E_{el}$$

Required final effective force = 9000 lb/ft (131,300 N/m) or a tendon spacing of one tendon per 3 ft (0.91 m) on center.

The average initial stress after friction and seating loss in this example = 191,600 psi (1321 MPa). Based on a tendon spacing of one tendon per 3 ft (0.91 m), the area of prestressing steel = 0.153/3 = 0.051 in.$^2$ (32.9 mm$^2$) per foot width of the slab.

$$K_c = 0.5 \text{ for post-tensioned members}$$

$$f_{pul} = (191,600 \times 0.153/3)/(12 \times 5 \text{ in.}) = 163 \text{ psi (1.12 MPa)}$$

$$\Delta f_{ES} = (0.5)(28,500,000 \text{ psi})(163 \text{ psi})/(3,122,000 \text{ psi}) = 744 \text{ psi (5.13 MPa)}$$

8.2.5 Creep—The long-term losses such as creep, shrinkage, and relaxation are calculated using the simplified method described in Chapter 5.

$$\Delta f_{CR} = K_c E_c f_{pul} / E_{pul}$$

$$K_c = 1.6 \text{ (post-tensioned normal weight)}$$
\[ \Delta f_{CR} = (1.6)(28,500,000/3,605,000)(163 \text{ psi}) = 2062 \text{ psi (14.22 MPa)} \]

### 8.2.6 Shrinkage
\[ \Delta f_{SH} = (8.2 \times 10^{-6})K_{sh}E_p(1 - 0.06I/S)(100 - RH) \]
\[ K_{sh} = 0.85 \]
\[ I/S = 2.5 \text{ in. (63.5 mm)} = (8.2 \times 10^{-6})(0.85)(28,500,000)(1 - 0.06(2.5))(100 - 80) = 3377 \text{ psi (23.28 MPa)} \]

### 8.2.7 Relaxation
\[ \Delta f_{RE} = [K_{.e} - J(\Delta f_{ES} + \Delta f_{CR} + \Delta f_{CS})]C \]

From Table 5.4:
\[ K_{.e} = 5000 \text{ psi (34.47 MPa)} \]
\[ J = 0.04 \]
Average initial tendon stress \( f_{pi} = 191,600 \text{ psi (1321 MPa)} \)
\[ f_{pi}/f_{pu} = 191,600 \text{ psi}/270,000 \text{ psi} = 0.71 \]
\[ C = (0.71/0.21)(0.71/0.9 - 0.55) = 0.807 \]
\[ \Delta f_{RE} = [5000 - 0.04(3377 + 2062 + 744)](0.807) = 3835 \text{ psi (26.44 MPa)} \]

### 8.2.8 Total long-term losses—Total losses are:
\[ \Delta f_T = \Delta f_{ES} + \Delta f_{CR} + \Delta f_{SH} + \Delta f_{RE} = 744 + 2062 + 3377 + 3835 = 10,000 \text{ psi (69.0 MPa)} \]

Final effective tendon stresses after all losses are calculated by subtracting the total long-term losses from the tendon stress after anchoring (Fig. 8.2.3). The final effective force in the tendon per foot is \((191,600 - 10,000)(0.153/3) = 9260 \text{ lb/ft (135.1 kN/m)}\) compared to required 9000 lb/ft (131.4 kN/m).

### 8.3—Post-tensioned beam with bonded tendons

Example 8.3 presents calculation of prestress losses in a two-span post-tensioned beam with one 12-strand bonded tendon with 1/2 in. (12.7 mm) strands in corrugated metal ducts. The initial losses are calculated based on the methods in Chapter 4 and long-term losses are calculated using the simplified method described in Chapter 5.

#### 8.3.1 Given and problem statement—This problem considers a continuous two-span post-tensioned beam with 60 and 30 ft spans (18.3 m and 9.1 m) (Fig. 8.3.1) and a tributary bay width of 18 ft (5.49 m). The beam size is 18 x 34 in. (457 x 864 mm), assuming an effective flange width of 98 in. (2489 mm) and the slab is a cast-in-place 5 in. (127 mm) thick post-tensioned slab.

![Fig. 8.3.1—Beam dimensions and prestressing.](image_url)

Concrete properties: Normalweight
\[ f_{c'f} = 3000 \text{ psi (20.68 MPa) (at transfer)} \]
\[ f_{c'} = 5000 \text{ psi (34.47 MPa)} \]
\[ w = 150 \text{ lb/ft}^3 (2403 \text{ kg/m}^3) \]
\[ E_o = 3,122,000 \text{ psi (21.530 MPa)} \]
\[ E_c = 4,030,000 \text{ psi (27.790 MPa)} \]
Loads:
\[
SW = (18 \text{ ft} \times \frac{5}{12} + 18\text{ ft} \times \frac{34 - 5}{12}) (150 \text{ lb/ft}^3) = 1670 \text{ lb/ft} (24.37 \text{ kN/m})
\]
\[
SDL = 10 \text{ lb/ft}^2 \times 18 \text{ ft} = 180 \text{ lb/ft} (2.63 \text{ kN/m}) \text{ (assume 10 lb/ft}^2 [0.048 \text{ kN/m}^2] \text{ superimposed sustained load after beam is placed in service)}
\]
\[
RH = 60 \text{ percent}
\]

Section properties:
\[
\text{Effective flange width} = 40 \text{ in.} + 18 \text{ in.} + 40 \text{ in.} = 98 \text{ in.} (2.49 \text{ m})
\]
\[
\text{T-beam cross section} = 5 \times 98 + 18 \times (34 - 5) = 1012 \text{ in.}^2 (0.653 \text{ m}^2)
\]
\[
\text{Moment of inertia of T-beam} = 110,648 \text{ in.}^4 (0.0461 \text{ m}^4)
\]
\[
\text{Distance of neutral axis from top} = 11.27 \text{ in.} (286.3 \text{ mm})
\]
\[
\text{Distance of neutral axis from bottom} = 22.73 \text{ in.} (577.3 \text{ mm})
\]

Tendon properties:
Low-relaxation, 12-strand bonded system
\[
\text{Strand diameter} = \frac{1}{2} \text{ in.} (12.7 \text{ mm})
\]
\[
\text{Strand area} = 0.153 \text{ in.}^2 (98.7 \text{ mm}^2)
\]

Tendon area:
\[
A_{ps} = 12 \times 0.153 \text{ in.}^2 = 1.836 \text{ in.}^2 (1184 \text{ mm}^2)
\]
\[
E_p = 28,500,000 \text{ psi (196.5 GPa)}
\]
\[
f_{pu} = 270,000 \text{ psi (1862 MPa)}
\]
\[
\text{Jacking stress} = 0.8 \times 270,000 \text{ psi} = 216,000 \text{ psi (1489 MPa)}
\]
\[
\mu = 0.25 \text{ (rad}^{-1}) \text{ (Table 4.4.2)}
\]
\[
\kappa = 0.0002 \text{ ft}^{-1} (0.000656 \text{ m}^{-1})
\]
\[
\text{Anchor set} = 0.375 \text{ in.} (9.53 \text{ mm}) \text{ (assumes jack without power seating)}
\]

Problem statement: Determine friction, anchor set, elastic, creep, shrinkage and relaxation losses at tendon location A and B as shown in Fig. 8.3.1.

8.3.2 Friction losses and elongation—Similar to Section 8.2.2, the tendon eccentricity and the angular deviation at each parabolic vertex is determined to compute \( e^{\mu (\theta + k \delta)} \). Multiplying by the jacking stress gives the tendon stress at the end of each segment before anchor set. Elongations before anchor set are computed for each segment taking the average stress along the segment.
Table 8.3.2—Friction and anchor set losses, and elongations

| Vertex | $e_i$, in. | $|e_i - e_j|$, in. | $x_i$, ft distance from stressing end | $|x_i - x_j|$, in. | $\theta_i$, rad segment angular deviation | $\alpha_i$, rad angular stress in percent of stressing end | $\exp(-k \times \theta_i)$ stress in percent of stressing end | Stress before set, ksi | Stress after set, ksi | Elongation, in. |
|--------|-----------|------------------|-------------------------------|------------------|--------------------------|---------------------------------|---------------------------------|----------------|----------------|---------------|
| Stressing end | 1 | 22.75 | 0.00 | 0.0 | 0.0 | 0.0 | 0.0000 | 0.0000 | 1.000 | 216.0 | 175.4 | 0.00 |
| | 2 | 18.80 | 3.95 | 6.0 | 6.0 | 0.109 | 0.1093 | 0.972 | 209.9 | 181.5 | 0.54 |
| Midspan 1 / A | 3 | 3.00 | 15.80 | 30.0 | 24.0 | 0.109 | 0.2186 | 0.941 | 203.3 | 188.1 | 2.63 |
| | 4 | 25.40 | 22.40 | 54.0 | 24.0 | 0.154 | 0.3729 | 0.901 | 194.7 | 194.7 | 4.64 |
| Support 2 / B | 5 | 31.00 | 5.60 | 60.0 | 6.0 | 0.154 | 0.5272 | 0.866 | 187.1 | 187.1 | 5.12 |
| | 6 | 29.00 | 2.00 | 63.0 | 3.0 | 0.111 | 0.6379 | 0.842 | 181.9 | 181.9 | 5.35 |
| Midspan 2 | 7 | 21.00 | 8.00 | 75.0 | 12.0 | 0.111 | 0.7485 | 0.817 | 176.5 | 176.5 | 6.26 |
| | 8 | 22.40 | 1.40 | 87.0 | 12.0 | 0.019 | 0.7680 | 0.811 | 175.2 | 175.2 | 7.15 |
| Support 3 | 9 | 22.75 | 0.35 | 90.0 | 3.0 | 0.019 | 0.7807 | 0.807 | 174.2 | 174.2 | 7.37 |

Stress before set, ksi: 1489.3, 1209.4, 0.00
Stress after set, ksi: 1447.4, 1251.3, 1.37
Elongation, in.: 1410.7, 1297.1, 6.67

Figure 8.3.2 shows that a linear friction approximation is not considered for this tendon; rather, the specified tendon curvature is used for more accuracy.

**Fig. 8.3.2—Tendon tension diagram.**

8.3.3 Anchor set losses—Anchor set losses are computed similarly to those in 8.2.3. For this example,

Anchor set length = 51.1 ft (15.6 m)
Maximum stress = 195,700 ksi (1349 MPa)
Average initial stress = 184,500 ksi (1272 MPa)

8.3.4 Computing concrete stresses—A structural analysis is performed to obtain moments presented in Table 8.3.4 assuming uncracked sections.
Table 8.3.4—Moments in beam

<table>
<thead>
<tr>
<th></th>
<th>Point A</th>
<th>Point B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Midspan 1</td>
<td>Support 2</td>
</tr>
<tr>
<td>Post-tensioning (PT) moment kip-ft (kN-m)</td>
<td>-454.36 (-616.03)</td>
<td>471.5 (639.27)</td>
</tr>
<tr>
<td>SW moment kip-ft (kN-m)</td>
<td>469.87 (637.06)</td>
<td>-563.25 (-763.67)</td>
</tr>
<tr>
<td>SDL moment kip-ft (kN-m)</td>
<td>50.65 (68.67)</td>
<td>-60.71 (-82.31)</td>
</tr>
</tbody>
</table>

To compute creep and elastic shortening losses, $f_{cpe}$, $f_e$, and $f_{cub}$ stresses are needed at both points: $f_{cpe}$ is concrete stress at center of gravity of tendon due to $P$; $f_e$ is concrete stress at center of gravity of tendon due to member self-weight; $f_{cub}$ is concrete stress at center of gravity of tendon due to all superimposed permanent dead loads; and $f_{ceu}$ is net concrete stress at center of gravity of tendons at transfer.

As a simplification, $f_{ceu}$ is calculated with the average tendon stress after transfer. It could also be calculated with the stress determined for each location (given in Table 8.3.2).

At Point A:

$$f_{cpe} = \frac{184,500 \text{ psi} \times 1.836 \text{ in.}^2}{1012 \text{ in.}^2} = 184.5 \text{ psi}$$

(Note: compression stress is positive and tension stress is negative)

$$f_e = \frac{469.870 \times 12 \times (-19.73 \text{ in.})}{110,648 \text{ in.}^4} = -1005 \text{ psi}$$

$$f_{cub} = \frac{50,650 \times 12 \times (-19.73 \text{ in.})}{110,648 \text{ in.}^4} = -108 \text{ psi}$$

$$f_{ceu} = K_{cir} \times f_{cpe} - f_e = 1.0 \times 1307 \text{ psi} - 1005 \text{ psi} = 302 \text{ psi}$$

At Point B:

$$f_{cpe} = \frac{184,500 \text{ psi} \times 1.836 \text{ in.}^2}{1012 \text{ in.}^2} = 184.5 \text{ psi}$$

$$f_e = \frac{563.250 \times 12 \times (-8.27 \text{ in.})}{110,648 \text{ in.}^4} = -505 \text{ psi}$$

$$f_{cub} = \frac{-60.710 \times 12 \times (8.27 \text{ in.})}{110,648 \text{ in.}^4} = -54 \text{ psi}$$

$$f_{ceu} = K_{cir} \times f_{cpe} - f_e = 1.0 \times 758 \text{ psi} - 505 \text{ psi} = 253 \text{ psi}$$

8.3.5 Elastic shortening—The elastic shortening loss is calculated using the simplified method described in Section 4.5.

$$\Delta f_{Eds} = K_e E_{pds} f_{cu} E_{es} - K_e = 0$$

because all strands are tensioned simultaneously

At Point A:

$$\Delta f_{Eds} = (0)(28,500,000 \text{ psi})(302 \text{ psi})/(3,122,000 \text{ psi}) = 0 \text{ psi}$$

At Point B:

$$\Delta f_{Eds} = (0)(28,500,000 \text{ psi})(253 \text{ psi})/(3,122,000 \text{ psi}) = 0 \text{ psi}$$

8.3.6 Creep—The long-term losses such as creep, shrinkage and relaxation are calculated using the simplified method described in Chapter 5. For bonded tendons, creep shortening losses are computed at each point.
\[ \Delta f_{pCR} = K_{cr}(E_{po}E_{c})(f_{oc} - f_{cd}) \]

\( K_{cr} = 1.6 \) (post-tensioned normalweight)

At Point A:

\[ \Delta f_{pCR} = (1.6)(28,500,000/4,030,000)(302 \text{ psi} - 108 \text{ psi}) = 2195 \text{ psi} \]

15.13 MPa

(caution is warranted with signs; \( f_{cd} \) is subtracted if in tension)

At Point B:

\[ \Delta f_{pCR} = (1.6)(28,500,000/4,030,000)(253 \text{ psi} - 54 \text{ psi}) = 2252 \text{ psi} \]

15.53 MPa

8.3.7 Shrinkage

\[ \Delta f_{pSh} = (8.2 \times 10^{-6})K_{sh} E_{po}(1 - 0.06V/S)(100 - RH) \]

\[ K_{sh} = 0.85 \]

\[ V/S = 1012 \text{ in}^{2}/(2 \times 98 + 2 \times 29) = 3.984 \text{ in.} \]

101.2 mm

At both points:

\[ \Delta f_{pSh} = (8.2 \times 10^{-6})(0.85)(28,500,000)[1 - 0.06(3.984)][100 - 60] = 6046 \text{ psi} \]

41.69 MPa

8.3.8 Relaxation

\[ \Delta f_{pRE} = \left[ K_{ir} - J(\Delta f_{pSh} + \Delta f_{pCR} + \Delta f_{pES})\right]C \]

Relaxation losses depend on creep, elastic shortening and shrinkage losses, and are calculated at both points.

From Table 5.4:

\[ K_{ir} = 5000 \text{ psi} \]

34.47 MPa

\[ J = 0.04 \]

Average initial tendon stress \( f_{pi} = 184,500 \text{ psi} \)

1272 MPa

\[ f_{po}/f_{pi} = 184,500 \text{ ksi}/270,000 \text{ psi} = 0.683 \]

\[ C = [0.683/0.21][0.683/0.9 - 0.55] = 0.681 \]

At Point A:

\[ \Delta f_{pRE} = [5000 - 0.04(6046 + 2195 + 0)](0.681) = 3181 \text{ psi} \]

21.93 MPa

At Point B:

\[ \Delta f_{pRE} = [5000 - 0.04(6046 + 2252 + 0)](0.681) = 3179 \text{ psi} \]

21.92 MPa

Difference is negligible in this case; relaxation losses can be computed with an average value of creep shortening losses.

8.3.9 Total long-term losses

At Point A:

\[ \Delta f_{pT} = \Delta f_{pES} + \Delta f_{pCR} + \Delta f_{pSh} + \Delta f_{pRE} = 0 + 2195 + 6046 + 3181 = 11,422 \text{ psi} \]

78.8 MPa

At Point B:

\[ \Delta f_{pT} = \Delta f_{pES} + \Delta f_{pCR} + \Delta f_{pSh} + \Delta f_{pRE} = 0 + 2252 + 6046 + 3179 = 11,477 \text{ psi} \]

79.1 MPa
Because long-term losses at each point of interest are close, it is sufficient to subtract the mean long-term losses (11,450 psi [78.9 MPa]) from the tendon stresses after anchor set to compute tendon stresses after all losses along the tendon (Eq. 8.3.2). The final effective force in the beam assuming the mean long-term loss of 11,450 psi (78.9 MPa) = (184,500 – 11,450) x 1.836 in.² = 317,700 lb (1431.2 kN).

8.4—Example with heat of hydration during casting

The effect on prestress loss of the heat of hydration in pretensioned concrete members is presented with this example. The method presented in 4.2.4 is used in the example.

8.4.1 Description of lab beam and instrumentation—Small test beams were cast in the laboratory to investigate transfer and development length of prestressing strands in lightweight concrete (Cross 2012). Instrumentation included load cells on each of the three strands outside of the beam, which were monitored from the stressing operations until just before cutting the strands. In addition, a vibrating wire gauge was placed at the centroid of the strand pattern at midspan each of the test beams. Each vibrating wire gauge contains a thermistor, so temperatures in the beam were recorded from just before casting the concrete until just before testing the beams several months later. Figure 8.4.1a presents the temperature history and Fig. 8.4.1b presents the average stress calculated from the loads recorded by the load cells on the three strands from before casting to just before transfer. These beams were not steam cured, so they were left in the stressing beds for 7 days before release strength was achieved.

Figure 8.4.1c shows the cross section of the beam. There were two 24 ft (7.32 m) long beams cast in a 60 ft (18.3 m) long bed. The following calculations determine the changes in stresses in the strand and in the concrete during the heating and cooling cycles.

**Fig. 8.4.1a—Temperature history for beam.**

**Fig. 8.4.1b—Average stress in strands from stressing to just before transfer based on load cell.**

**Fig. 8.4.1c—Cross section of beam in lab study.**

8.4.2—Calculations of stress changes during curing

Given information for this problem is:

- Temperature rise = 48°F (26.7°C) (Fig. 8.4.1a)
- Temperature fall = 58°F (32.2°C) (Fig. 8.4.1a)
Free strand length = 12 ft = 144 in. (3658 mm)
Strand length in two beams = 48 ft = 576 in. (14,630 mm)
Coefficient of thermal expansion of steel = 6 µE/°F (10.8 µE/°C)
Area of strand = 3 × 0.217 in.² = 0.651 in.² (420 mm²)
Modulus of strand = 28,500,000 psi (196,500 MPa)
Coefficient of thermal expansion of concrete = 5 µE/°F (9.0 µE/°C)
Net area of concrete = 272 in.² (175,483 mm²)
Net moment of inertia of concrete = 15,194 in.⁴ (6.324 × 10⁹ mm⁴)
Modulus of concrete = 4,800,000 psi (33,095 MPa)

During the temperature rise, it is assumed that the concrete has no stiffness and there is no bond between the concrete and the steel. It is assumed that the temperature of the strands is the same as that of the concrete along the full length. Therefore, the change in the stress in the prestress is assumed to be

\[
\Delta f_{\text{temp rise}} = \Delta T \alpha_f L_f + \frac{\Delta P_{\text{free}} L_{\text{free}}}{A_f E_p} = 48 \times 6 \times 10^{-6} (28,500,000 \text{ psi}) = 8200 \text{ psi} (56.5 \text{ MPa})
\]

As is seen in Fig. 8.4.1b, this is considerably less than the 15,000 psi (103 MPa) loss recorded by the load cells. This can partly be explained by higher relaxation losses at elevated temperatures, which is discussed in Rostásy et al. (1991).

Next, the calculation of stress changes due to cooling is performed. First three changes in force are defined: \( \Delta P_{\text{free}} \) is change in the force in the free strand length, lb (N); \( \Delta P_c \) is change in force in the concrete cross section, lb (N); and \( \Delta P_{\text{ps}} \) is change in force in the strand within the concrete beam, lb (N).

The sum of the beam forces are equal to the change in force in the free strand length:

\[
\Delta P_{\text{ps}} + \Delta P_c + \Delta P_{\text{free}} = 0
\]

The total change in length of the free strand and the beams are equal to zero. The temperature was assumed to be constant along the strand to simplify the analysis. The individual changes are calculated as:

\[
\Delta_{\text{free}} = -\Delta T \alpha_f L_f + \frac{\Delta P_{\text{free}} L_{\text{free}}}{A_f E_p}
\]

\[
\Delta_c = -\Delta T \alpha_c L_{\text{mean}} + \frac{\Delta P_{\text{ps}} L_{\text{ps}}}{A_c E_c} + \frac{\Delta P_c L_{\text{ps}}}{I_c E_c}
\]

\[
\Delta_{\text{ps}} = -\Delta T \alpha_c L_{\text{mean}} + \frac{\Delta P_{\text{ps}} L_{\text{ps}}}{A_p E_p}
\]

Two compatibility equations are written:

\[
\Delta_P = \Delta_c
\]

\[
\Delta_{\text{ps}} + \Delta_{\text{free}} = 0
\]

We now have a system of six equations and six unknowns. For this particular problem, the three equations for length change are:

\[
\Delta_{\text{free}} = -58 \times 6 \times 10^{-6} \times 144 + \frac{\Delta P_{\text{free}} 144}{0.651 \times 28,500} = -0.050 + \Delta P_{\text{free}} \times 0.0078
\]

\[
\Delta_c = -58 \times 5 \times 10^{-6} \times 576 + \frac{\Delta P_{\text{ps}} 576}{272 \times 4800} + \frac{\Delta P_c 13.9^2 \times 576}{15,194 \times 4800} = -0.167 + \Delta P_c \times 0.00197
\]

\[
\Delta_{\text{ps}} = -58 \times 6 \times 10^{-6} \times 576 + \frac{\Delta P_{\text{ps}} 576}{0.651 \times 28,500} = -0.200 + \Delta P_{\text{ps}} \times 0.031
\]

Solving the equations simultaneously results in the following:

\[
\Delta P_{\text{free}} = 22,800 \text{ lb (101,400 N) (tension)}
\]

\[
\Delta P_c = 20,400 \text{ lb (90,700 N) (tension)}
\]
\[ \Delta P_{ps} = 2400 \text{ lb (10,700 N) (tension)} \]
\[ \Delta_{\text{free}} = 0.127 \text{ in. (3.23 mm) (lengthening)} \]
\[ \Delta_{e} = \Delta_{ps} = -0.127 \text{ in. (-3.23 mm) (shortening)} \]

The stress increase in the free strand length would be 35,000 psi (241.3 MPa). According to Fig. 8.4.1b, the stress increase due to cooling was approximately 28,000 psi (193.0 MPa), so the method overestimates the change. The stress increase of the strand within the beam is calculated to be 3600 psi (24.8 MPa), compared to a measured change of approximately 3000 psi (20.7 MPa) (Fig. 8.4.3). Before release, the concrete has a calculated stress of 336 psi (2.3 MPa) in tension at the level of the center of gravity of the strand.

**Stress in concrete prior to transfer**

\[
\text{stress in concrete prior to transfer} = \frac{20.4 \text{ kip}}{272 \text{ in.}^2} + \frac{20.4 \text{ kip}(13.9 \text{ in.})^2}{15,194 \text{ in.}^4} = 336 \text{ psi (2.32 MPa)}
\]

Using the original jacking force and the transformed cross-sectional properties, the compressive stress in the concrete at the level of the strand is:

\[
f_c = \frac{-189 \text{ ksi} \times 0.651 \text{ in.}^2}{276 \text{ in.}^2} + \frac{-189 \text{ ksi} \times 0.651 \text{ in.}^2 \times (13.7 \text{ in.})^2}{15,934 \text{ in.}^4} + \frac{246 \text{ in.-kip} \times 13.7 \text{ in.}}{15,934 \text{ in.}^4} = -1685 \text{ psi (11.6 MPa)}
\]

The compressive stress in the concrete at transfer can be calculated using the stress that at the time the concrete initially bonded to the concrete. All changes occurring thereafter are recoverable.

\[
\text{stress in strand bonding} = 189 \text{ ksi} - 8.2 \text{ ksi} = 180.8 \text{ ksi}
\]

\[
f_c = \frac{-180.8 \text{ ksi} \times 0.651 \text{ in.}^2}{276 \text{ in.}^2} \times \frac{-180.8 \text{ ksi} \times 0.651 \text{ in.}^2 \times (13.7 \text{ in.})^2}{15,934 \text{ in.}^4} + \frac{246 \text{ in.-kip} \times 13.7 \text{ in.}}{15,934 \text{ in.}^4} = -1600 \text{ psi (11.03 MPa)}
\]

The apparent compressive stress in the concrete at release would be 336 psi + 1600 psi = 1936 psi (2.32 MPa + 11.03 MPa = 13.35 MPa) due to the removal of the restraint forces at transfer. Whereas, the actual stress in the concrete of 1600 psi represents a 5 percent reduction due to the loss of prestress between initial tensioning and bond. As mentioned previously, some simplifying assumptions are made in this analysis; however, it provides a general idea of the stress changes in the steel and concrete due to heating and cooling of the beam.

**8.4.3 Comparisons of model and measurements**—The graphs (Fig 8.4.1b and 8.4.3) confirm the behavior predicted by the model. As the concrete heats up, strand stress decreases as shown in Fig. 8.4.1b. The decrease, measured by load cells, is larger than would be expected. As calculated, the expected change in stress is 8200 psi (56.5 MPa), but the change measured with the load cells is approximately 15,000 psi (103.4 MPa)—almost twice the expected change. Part of the loss could be relaxation; tests have shown that relaxation can be significantly increased by elevated temperature (Rostásy et al. 1991).

As the concrete cools, the strand stress as measured by both the load cells and the internal gauges increases. Figure 8.4.3 shows the strand stress as measured by the vibrating wire gauge embedded in the concrete. As the model predicts, the increase in strand stress external to the beam is greater than that inside the beam. Assuming a concrete modulus of elasticity of 4,800,000 psi (33 GPa), the model predicted an increase of 35,000 psi (241.3 MPa) in the strand external to the beam, while the measured increase was approximately 28,000 psi (193.0 MPa). The model predicted an increase in strand stress within the concrete to be approximately 3600 psi (24.8 MPa), which is similar to the measurement with the vibrating wire gauges (VWGs).

**8.4.4 Conclusions from temperature example**—In conclusion, the effect of temperature changes during casting can be examined qualitatively. A quantitative evaluation is difficult due to the uncertainty in determining the early-age concrete properties, the influence of temperature on relaxation, and the effects of formwork restraint. Thermal effects are considered to be partly responsible for measurements of elastic shortening, which are higher than theoretical. Because the net influence on the beam is small, for most applications it is acceptable to ignore the thermal effects during casting.
Fig. 8.4.3—Strand stress based on vibrating wire gauges (VWGs).
CHAPTER 9—REFERENCES

ACI committee documents and documents published by other organizations are listed first by document number, full title, and year of publication followed by authored documents listed alphabetically.

American Concrete Institute

ACI 209R-92(08)—Prediction of Creep, Shrinkage, and Temperature Effects in Concrete Structures
ACI 209.1R-05—Report on Factors Affecting Shrinkage and Creep of Hardened Concrete
ACI 209.2R-08—Guide for Modeling and Calculating Shrinkage and Creep in Hardened Concrete
ACI 318-63—Building Code Requirements for Reinforced Concrete
ACI 318-83—Building Code Requirements for Reinforced Concrete
ACI 318-99—Building Code Requirements for Structural Concrete and Commentary
ACI 318-08—Building Code Requirements for Structural Concrete and Commentary
ACI 318-11—Building Code Requirements for Structural Concrete and Commentary
ACI 363R-92(97)—Report on High Strength Concrete
ACI 435R-95(00)—Control of Deflection in Concrete Structures (Appendix B added 2003)

ASTM International

ASTM A416/A416M-12—Standard Specification for Steel Strand, Uncoated Seven-Wire for Prestressed Concrete

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EN 13670-09—Execution of Concrete Structures

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PTI C30.4-07—Training and Certification of Field Personnel for Bonded Post-Tensioning
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Comité Euro-International du Béton (CEB), 1999, “Structural Concrete – Textbook on Behaviour, Design and Perfor-


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Storm, T., 2011, “Predicting Prestress Losses, Camber, and Deflection in Prestressed Concrete,” MS thesis, North Carolina State University, Raleigh, NC.

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